Implicit Riemannian Trust-Region Method for Symmetric Generalized Eigenproblems

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We have recently proposed a manifold-based trust-region method to compute extreme eigenpairs of symmetric matrix pencils [3]. The general optimization method had been shown [1, 2] to enjoy strong global and local convergence properties, which are inherited by its "brute force" application to the extreme eigenproblem. However, the trust-region mechanism, which restricts the norm of the update in an adaptive way, has some inherent inefficiencies: when the trust-region is too large, valuable time may be spent computing an update that ends up being rejected; when the trust-region is too small, computational effort may be wasted computing updates that are exceedingly conservative. It may take the adaptive process several iterates to bring the trust-region radius to an appropriate size. In this work, we show how the particular structure of the eigenproblem can be exploited to remedy this drawback. We present an analysis that provides us knowledge of the model fidelity at every step of the inner iteration, making the standard, explicit trust-region mechanism obsolete, while preserving the convergence properties. We also show how the stopping criterion of the outer iteration can be monitored in the inner iteration, in order to prevent the final call to the inner iteration from performing more work than is needed.

References

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[3] ———, *A truncated-CG style method for symmetric generalized eigenvalue problems*. J. Comput. Appl. Math., to appear, 2006.