Modeling Credit Risk:

Currency Dependence in Global Credit Markets

Alec Kercheval

Associate Professor
Dept of Mathematics
Florida State University
Tallahassee, FL 32306-4510
Phone: 850-644-8701
Fax: 850-644-4053
kercheval@math.fsu.edu

Lisa Goldberg

Director
Fixed Income Research
Barra
2100 Milvia Street
Berkeley, CA 94709
Phone: 510-643-4601
Fax: 510-848-0954
lrg@barra.com

Ludovic Breger*

Senior Consultant
Fixed Income Research
Barra
2100 Milvia Street
Berkeley, CA 94709
Phone: 510-643-4613
Fax: 510-848-0954
ludovic.breger@barra.com

ABSTRACT

We investigate credit spreads for euro-, sterling-, and US dollar-denominated credit instruments relative to their local swap curves, and show that monthly spread changes are strongly currency-dependent during the study period May 1999 to May 2001. Sector-by-rating factor returns are at best weakly correlated across currencies, and U.S. dollar spread return volatilities are generally higher than the other two by a factor of two or three. This is contrary to what would be expected from covered interest arbitrage. We conclude that credit factor risk models in each of the three markets should be estimated separately, and risk forecasting models using a single set of spread factors to cover more than one of these markets will suffer from poor accuracy.

* Corresponding author

INTRODUCTION

The global credit market consists of bonds exposed to credit risk relative to domestic treasury issues. These include corporate, agency, foreign sovereign, and supranational bonds, as well as credit derivatives such as default swaps and credit spread products. By all accounts, this market has experienced rapid growth in the past few years [O'Kane, 2000]. Bond portfolios are thus increasingly likely to include bonds other than domestic treasuries, and so are increasingly exposed to credit risk. There is therefore growing demand for more detailed models capturing the behavior of credit spreads. This study addresses the development of factor models for credit spread changes in the euro, sterling, and US dollar markets.

An important question in developing such models is whether bond credit spread changes can safely be assumed market-independent. For example, if Toyota issues both sterling- and euro-denominated bonds, one might expect the credit risk of the bonds to depend only on Toyota's creditworthiness and not on the currency in which the bond was issued. Therefore it would be reasonable to suppose that credit spread returns (i.e. spread changes) in the two markets should be roughly the same, with total yield changes explained entirely by the behavior of the underlying swap curves with respect to which the spread is calculated.

Reinforcing this intuition are interest arbitrage arguments (see Interest Arbitrage section), showing that corresponding spread changes across efficient markets should be highly correlated. If this were true, the task of constructing a multi-market credit risk model would be simplified since spread volatilities could be estimated from the universe of bonds across all markets. We would enjoy lower estimation noise and would be able to estimate a great number of different factor returns. Surprisingly, the data shows that, at least over the study period, spread changes have only weak correlation across the three markets. This failure of interest arbitrage to determine spread relationships means that credit risk factor models need be built independently in each market.

RISK MODEL AND DATA

We model credit risk using a multi-factor approach, as follows. Starting with a pool of investment grade bonds denominated in a single currency, we partition the pool into buckets comprised of all bonds sharing the same rating and sector classification (see Table 1). These buckets define our factors: Financial AAA, Utility A, etc.

Factor returns are defined in the following way: each month, as of the last business day of the month, we look at the one-month spread change for each bond in a bucket, and then compute a duration-weighted average spread change (i.e. spread return) across the bucket (with some outlier rejection scheme). This average spread return is our factor return, one for each of N sector-by-rating factors. Each bond is exposed only to the factor corresponding to its sector and rating; the value of the exposure is the spread duration of that bond. For a universe of K bonds, this gives us a linear model of asset returns

$$R = XF + \Psi$$

where X is the K by N matrix of bond exposures to the factors, F is the vector of N factor returns, and Ψ is the vector of specific returns not explained by common factors. We assume that factor and specific returns are uncorrelated so that the K by K covariance matrix of asset returns can be expressed as:

$$C = X\Phi X^T + S$$

where Φ is the covariance matrix of factor returns, and S is the (diagonal) covariance matrix of specific returns.

In this study, we restrict our attention to the common factor portion Φ, which captures the market component of the risk of credit instruments. (A rule of thumb is that the common factor risk dominates specific risk for investment grade bonds.)

Our euro and sterling data consists of bond prices for the constituents of the EuroBIG and EuroSterling investment grade indices [Salomon Smith Barney, 1999] supplied by Salomon Smith Barney for the 25-month period May 1999 to May 2001. The US data comprise the investment grade component of the Merrill Lynch US Corporate/Government Master Index [Merrill Lynch, 2000].

Sectors, ratings, and the typical number of bonds exposed to each factor are shown in Table 1. Factors were excluded when fewer than five bonds were available to estimate a factor return. On average, the study made use of about 500 euro-denominated bonds, 200 sterling denominated bonds, and 3700 US dollar-denominated bonds.

Our question may now be restated as follows: do returns for factors common to any two of the three markets behave similarly? For example, does the euro Financial AA factor return roughly track the sterling Financial AA factor return? We show below that the answer is no.

The remainder of this paper is organized as follows. In “Interest arbitrage” we describe the arbitrage arguments leading us to expect high factor return correlations across currencies. In “Factor returns”, we present the results of our computation of monthly factor returns over the study period. After a quick look at volatility levels, we examine various correlations among factors to try to understand how different segments of the markets are related. We examine rating correlations within a sector, sector correlations within a rating, and correlations across markets. We also resolve an apparently anomalous correlation between the euro Utility and sterling Industrial sectors. In "Statistical confidence levels", we address the statistical significance of the conclusions drawn from the data in "Factor returns". In "A search for financial explanations", we examine (and reject) some possible explanations for the lack of cross-market correlation: issuer-specific factors arising from the relatively small proportion of issuers common to more than one market, and the possibility that non-sector/rating factors such as coupon, duration, maturity, or amount outstanding may explain the spread return differences across markets.

INTEREST ARBITRAGE

In a perfect market, covered interest arbitrage implies a definite relationship between the spreads of bonds issued in different currencies but that are otherwise equivalent. For convenience, we summarize this in the following proposition.

For concreteness, suppose our two currencies are dollars and euros, and XYZ company issues one year pure discount bonds (PDBs) in both. Let:

r_d (r_e) denote the annually compounded risk-free one year spot rate in dollars (respectively, euros)

s_d (s_e) denote the spread of an XYZ one year PDB in dollars (respectively, euros) at issue

Proposition 1: Given r_d, r_e and s_d in a perfect market the requirement of no arbitrage implies that the quantity s_e is determined by the relation

$$s_e = \left(\frac{1+r_e}{1+r_d}\right) s_d \qquad (1.1)$$

Proof:

Let X_0 denote the spot exchange rate (1 dollar = X_0 euros), and X_F denote the 1 year forward exchange rate. The requirement of no arbitrage determines the forward exchange rate as follows.

Today, borrow one dollar at rate r_d, exchange to X_0 euros, and lend that amount at rate r_e. Simultaneously enter a one year forward contract to exchange $X_0\left(1+r_e\right)$ euros into dollars at exchange rate X_F.

One year later, after exchanging back to dollars at the contract rate, you have $\frac{X_0(1+r_e)}{X_F}$. If the net profit of this riskless arbitrage is to be zero, this must equal the amount owed on the dollar loan, $1+r_d$, which implies

$$X_F = \frac{X_0(1+r_e)}{1+r_d} \qquad (1.2)$$

Now for equation (1) short 1 dollar of XYZ dollar bonds, exchange to X_0 euros, purchase X_0 euros of XYZ euro bonds, and enter a forward contract to exchange $X_0\left(1+r_e+s_e\right)$ euros to dollars at exchange rate X_F.

After one year, the euro investment, after exchange to dollars, is worth

$$\frac{X_0(1+r_e+s_e)}{X_F}=\frac{(1+r_e+s_e)(1+r_d)}{(1+r_e)}$$

using (2).

For the net profit to be zero, this must be equal to the cost $(1+r_d+s_d)$ of repaying the short position. Solving for s_e yields the result.

Notice that the relationship between the two spreads s_d and s_e does not depend on the exchange rate, but does depend on the level of risk free rates in the two currencies. For example, if risk free rates remain constant but different in the two currencies, Proposition 1 implies that XYZ spreads will be unequal but perfectly correlated.

If risk free rates are not constant, we can analyze a small change in s_e in terms of changes in the other variables by differentiating:

$$ds_e=\left(\frac{1+r_e}{1+r_d}\right)ds_d+\frac{s_d}{1+r_d}\left[dr_e-\left(\frac{1+r_e}{1+r_d}\right)dr_d\right] \qquad (1.3)$$

When spread levels are moderate, as with investment grade bonds, and when risk free rate changes are not drastically larger than spread changes, the second term is small and we have the approximate relationship

$$ds_e \approx \left(\frac{1+r_e}{1+r_d}\right) ds_d \qquad (1.4)$$

This means under most circumstances we would expect spread changes (that is, spread returns) in the two currencies to be strongly correlated, of the same sign and very similar magnitude. In terms of a risk model based on spread returns as factors, equation (4) means the factor volatilities should be closely comparable across markets. The data shows, however, that quite the contrary is actually true, as we describe below.

FACTOR RETURNS

Volatility

Figure 1 shows a comparison of volatility forecasts as of May 31, 2001 for different factors common to the euro, sterling, and US dollar markets. These forecasts are computed as standard deviations in basis points per year based on historical monthly data from the study period that is exponentially weighted with a 24-month half-life, most recent returns weighted most strongly. US dollar volatilities are consistently higher than euro or sterling volatilities, frequently by a factor of two or three. Sterling

volatilities are sometimes closer to US dollar values, other times to euro

values. None of the markets is a good proxy for the others.

Correlations

The correlation matrix estimated from monthly returns for all three markets (52 factors in total) appears schematically in Figure 2. This 52-by-52 matrix contains 1,326 entries -- too many to display in a single table. Instead, this ``heat map" display gives a qualitative indication that the matrix has a block-diagonal character showing that correlations within markets are high while correlations between markets are low.

We investigate this in more detail below with a look at certain specific correlations. In Figure 3, we examine the monthly factor returns computed for the Financial AA factor in all three markets. Correlations between markets are weak at best. For example there are many months in which Financial AA spreads widen in one market and narrow in the others. (See Appendix A for returns for Supranational AAA and Industrial A factors.)

We look below at correlations from specified blocks of the full correlation matrix to show that in any single market, there is high return correlation between factors with a common sector or rating, but not across markets. Financial sector correlations are displayed in Table 2. Corresponding tables for the Industrial and Supranational sectors are collected in Appendix B, where they tell a similar story: individual markets are fairly unified, and returns across markets are relatively independent.

Table 3 displays correlations between factors having a common rating, with a similar pattern evident. Graphs of time series returns used to calculate these correlations are collected in Appendix C.

Anomalous cross-market correlation

Among the cross-market correlations in the full matrix of Figure 2, we noticed a curiously high value between euro Utility factors and sterling Industrials. This was striking especially since correlations between euro Industrials and sterling Industrials were low (See Table 4).

A close examination reveals that these correlations are not a statistical anomaly, but are are a consequence of the sector classification schemes used in the EuroBIG and EuroSterling indices. The Utility sector of EuroBIG has a large Telecommunications subsector, but the EuroSterling index has no Utility sector --- its numerous Telecommunications bonds reside in the Industrial sector. This inconsistent sector mapping explains the counterintuitive correlations displayed in Table 4.

We explored this by remapping the Telecommunications subsector of the EuroBIG Utility sector to the Industrial sector and then re-estimating the model. The result: the correlations between the euro Utility sectors and the sterling Industrial sectors dropped while the correlations between the euro and sterling Industrial sectors increased (see Table 5).

STATISTICAL CONFIDENCE LEVELS

The apparent independence of markets described above invites the question of whether our conclusions are statistically justified given the amount of data available to estimate our factor returns. Could our results be simply due to bad luck in the sample?

To investigate this, we test the null hypothesis that, for a given factor (say, Financial AA), the average return in a given month for euro-denominated bonds is equal to the average return for sterling-denominated bonds. That is, we assume that individual bond returns are independently drawn from a normal distribution with a common mean across markets, and ask for the probability, under that assumption, of seeing the data actually observed. To accomplish this, we compute a t-statistic for each month, and from that we compute a confidence level of rejection of the null hypothesis. Technically, our null hypothesis requires the use of a weighted t-statistic, because we assume that an indidividual bond's spread return is drawn from a normal distribution with variance proportional to the reciprocal of the bond's duration. This leads us to use the duration-weighted average return as the best unbiased linear estimator of the common mean, which is how the factor returns were actually calculated in our study (See Appendix D for a detailed discussion of the weighted t-statistic.)

Figure 4 shows, month by month, the confidence level of rejection of the null hypothesis. That is, in any month, 100 - (confidence level shown) is the probability that the observed data could have occurred by chance under the null hypothesis. Confidence levels above 95% indicate that we may safely assume the null hypothesis fails -- that is, that the mean spread return in each market is statistically different and therefore they should not be estimated together with a combined set of bonds).

A SEARCH FOR FINANCIAL EXPLANATIONS

We are suggesting that perceived credit risk, as reflected by how average spread levels change, is currency-dependent. A skeptic might suggest that in fact credit risk changes do not depend on currency but rather on other factors overlooked by our model, causing us to mis-attribute differences to currency.

Issuer-level data

For example, we notice that the lists of issuer names in each of our three indices have fairly small overlap, so the aggregate behavior of spreads in each market might be due to differing characteristics of the actual issuers in each market. This, however, appears not to be the case, as spreads for bonds even from the same issuer behave differently in different markets.

To show this, we found issuers active in more than one market and compared spread returns in each market. Table 6 lists four such issuers with a bond in each currency. In Table 7 we display the time series correlation of returns for each bond and for the three pairs of markets. Generally speaking, knowledge of credit spreads for an issuer in one market tells us very little about spreads for the same issue in another market.

Figure 5 shows this in more detail in the case of Toyota. In the same month, sterling Toyota spreads might widen at the same time as euro Toyota spreads are narrowing. Clearly, Toyota's default risk cannot be increasing and decreasing at the same time.

Other factors

One still might object that spread differences even for individual issuers can be explained by non-currency factors missing from the model. However, we were unable to find any factors with explanatory power.

We examined four possibilities: coupon (which may influence tax-related behavior), duration, maturity, and amount outstanding (as a proxy for liquidity). Examination of the data shows no strong trends linking these quantities to spread return. Amount outstanding showed a mild inverse correlation with spread level, but not spread return. Representative results are shown in Figure 6 for outstanding bonds as of May 31, 2001 issued by the European Investment Bank.

CONCLUSIONS

Our analysis shows that euro, sterling, and US dollar credit spreads have been largely uncorrelated during our study period of May 1999 to May 2001. Our immediate conclusion is that credit risk models need to be built separately for the three markets.

Given our findings above for individual issuers, it seems clear that monthly changes in spread-to-swap levels should not primarily be attributed to changes in perceived creditworthiness of the issuer. Our view is that credit spread changes in a given market instead primarily reflect changes in the average risk premium required by investors in that market for a given sector and credit rating. Causes for these fluctuations will be found in the overall economic and political conditions that influence investor confidence, and these conditions are somewhat separate for each of the three markets under study. This explanation is also consistent with our finding of high spread return correlations across sectors and ratings within a single market.

The failure of covered interest arbitrage to rigidly link credit risk across currencies indicates the presence of substantial frictions across credit market boundaries. The nature of these frictions would be an interesting topic for further study.

To the extent that markets become more unified in the future as a result of the globalization of investment portfolios, our conclusion of currency independence may have to be revised. For now, credit risk models should respect the tendency of euro, sterling, and US dollar-denominated credit spreads to go their own ways.

Appendix A

Supranational AAA and Industrial A monthly spread returns for the euro, sterling and U.S. dollar markets

As shown in Figures 7 and 8, there is no significant correlation of Supranational AAA and Industrial A factors across the euro, sterling, and U.S. dollar markets.

Appendix B

Correlations within and across markets for some sector-defined factors

In the Industrial sector, within-market correlations are high and cross-market correlations are near zero (Tables 8a–d). For Supranational AAA bonds, cross-market correlations are near zero (Table 9).

Appendix C

AA credit spread returns

Time-series analyses of AA credit spread returns for the euro, sterling and U.S. dollar markets show that, within each market and rating class, there are strong correlations across sectors (Figures 9a–c).

Appendix D

t-statistics for weighted means with application to risk factor models

In this appendix we describe how to generalize the standard t-statistic test for equality of the means when the assumption of a common variance no longer holds. We derive a formula for the generalized t-statistic (equation 3 below), which was used to compute the confidence levels reported in Figure 4. First, we describe the standard t-statistic. Suppose we have a sequence of independent samples from a normal distribution with mean μ_X and variance σ^2. Denote the sample values by $X_1, X_2, \ldots, X_n$. We use the notation $X_i \sim N(\mu X, \sigma^2)$, where $N(a, b)$ denotes the probability density function of a normal distribution with mean a and variance b.

The best (minimum variance) linear unbiased estimator of the mean μ is the sample mean

$$\bar{X} = \sum_{i=1}^{n} w_i X_i$$

If $Y_1, Y_2, \ldots, Y_m$ is another group of independent samples with $Y_i \sim N(\mu Y, \sigma^2)$, we could ask whether or not $\mu_X = \mu_Y$. We take the null hypothesis to be the statement that this equality is true.

Given our sample data, we cannot determine the truth or falsity of the null hypothesis, but we can determine the likelihood of the realized sample values assuming the null hypothesis. If this likelihood is small, we are justified in rejecting the null hypothesis.

To accomplish this, we may use the standard (Student's) t-statistic for equality of the mean:

$$T = \frac{\bar{X} - \bar{Y}}{\sqrt{\frac{nS_X^2 + mS_Y^2}{n+m-2}\left(\frac{1}{n} + \frac{1}{m}\right)}} \tag{1}$$

where

$$S_X^2 = \frac{1}{n}\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2$$

is the sample variance of *X*, and similarly for *Y.*

The random variable *T* has a *t*-distribution with $n + m$–2 degrees of freedom. Therefore, we can determine the probability that *T* is equal to or greater than the realized value, given $\mu_X = \mu_Y$. Typically, if this probability is below 5% or 1%, the null hypothesis is rejected.

In this paper we generalize the discussion to the case where the samples are drawn from distributions with a common mean but variances allowed to change from sample to sample:

$$X_i \sim N\left(\mu_X, \sigma_i^2\right)$$

In this case, the best linear unbiased estimate of the mean μX is the weighted average

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i \tag{2}$$

where

$$w_i = \frac{1/\sigma_i^2}{\sum_{j=1}^{n}\left(1/\sigma_j^2\right)}$$

Conversely, given positive weights w_i, i =1,...,n so that ,then the quantity in equation 2 is the best linear unbiased estimate of the mean provided that the samples are distributed as

$$X_i \sim N\left(\mu_X, \alpha_X / w_i\right)$$

for some constant $\alpha_X > 0$.

In either case, if

$$S_X = \sum_{i=1}^{n} w_i \left(X_i - \bar{X}\right)^2$$

is the weighted sample variance, and if we use similar notation for Y_i (with different weights w'_i allowed), then the corresponding formula for the *t*-statistic for equality of the weighted mean is

$$T = \frac{\bar{X} - \bar{Y}}{\sqrt{\frac{S_X/\alpha_X + S_Y/\alpha_Y}{n+m-2}}\sqrt{\alpha_X + \alpha_Y}} \tag{3}$$

Setting $w_i = 1/n$, $w_i = 1/m$, and $\alpha_Y = (n/m)\alpha_X$ reduces this expression to equation 1.

Note: T is independent of the scale of the pair (α_X, α_Y): if (α_X, α_Y) is replaced by $(k\alpha_X, k\alpha_Y)$ for some $k > 0$, the value of T is unchanged.

The weighted mean as a minimum variance estimator

If $X_1, X_2, \ldots, X_n$ is a random sample such that $X_i \sim N(\mu, \sigma_i^2)$, what is the minimum variance unbiased estimator of the mean? It is a weighted sum where greater weight is given to values coming from narrower distributions.

Let $X_i = \mu + e_i$ where e_i has mean 0 and variance σ_i^2. If

$$\bar{X} = \sum_{i=1}^{n} w_i X_i$$

is to be the minimum variance unbiased estimator of the mean μ, then we must solve for the weights w_i, minimizing the variance of X, subject to the constraint

$$\sum w_i = 1 \tag{4}$$

Because we are assuming that the variables e_i are independent, we have

$$\begin{aligned} E\left[\left(\bar{X}-\mu\right)^2\right] &= E\left[\left(\sum w_i e_i\right)^2\right] \\ &= \sum E\left[w_i^2 e_i^2\right] \\ &= \sum w_i^2 \sigma_i^2 \end{aligned}$$

The method of Lagrange multipliers to minimize this function subject to the constraint in equation 4 yields

$$w_i = \frac{1/\sigma_i^2}{\sum_{j=1}^{n}\left(1/\sigma_j^2\right)}$$

We obtain this weight if we set

$$\sigma_i^2 = \alpha / w_i$$

where α is any positive constant. This proves

Proposition 1 Let α be a positive constant. Suppose $w_1,\ldots,w_n$ are positive numbers satisfying $\Sigma w_i = 1$, and, for each i, X_i is a random variable with mean μ and variance α/w_i.

Then the minimum variance unbiased estimator of the mean μ is

$$\bar{X} = \sum_{i=1}^{n} w_i X_i$$

Establishing the weighted *t*-statistic

Recall that if a random variable V is the sum of the squares of $r > 0$ independent standard normal variables, then V is said to have a chi-squared distribution with r degrees of freedom.

The t-distribution with r degrees of freedom may be defined as the distribution of the random variable

$$T = \frac{W}{\sqrt{V/r}}$$

where W is a standard normal random variable, V has a chi-squared distribution with r degrees of freedom, and W and V are independent.

We need to show that the statistic defined in equation 3 has a t-distribution with n + m–2 degrees of freedom. We accomplish this with a sequence of lemmas in this section.

Standing assumptions: Let α_X and α_Y be fixed positive numbers. For i =1,...,n, and j =1,...,m, let

w_i and w'_j be positive numbers and X_i, Y_j independent random variables such that

- $\sum_{i=1}^{n} w_i = 1$ and $\sum_{j=1}^{m} w'_j = 1$

and

- for each i, j, $X_i \sim N(\mu, \alpha_X / w_i)$ and $Y_j \sim N(\mu, \alpha_Y / w'_j)$.

Notation:

- $\bar{X} = \sum w_i X_i$ and $\bar{Y} = \sum w'_j Y_j$
- $S_X = \sum w_i (X_i - \bar{X})^2$ and $S_Y = \sum w'_j (Y_j - \bar{Y})^2$

Lemma 1:

$\bar{X} \sim N(\mu, \alpha_X)$ *and* $\bar{Y} \sim N(\mu, \alpha_Y)$

Proof. A straightforward computation using the fact that a sum of independent normals is normal and variances add.

Lemma 2:

X(mean), *Y(mean)*, S_X, and S_Y are mutually independent.

Proof. Clearly *X(mean)* and *Y(mean)* are independent, and similarly for S_X and S_Y .We show that *X(mean)* is independent of S_X , and the same argument works for S_Y. The argument is a direct generalization of the proof for the equal weighted case found, for example, in Hogg and Craig (1995), which we include here for the reader's convenience.

Write $\alpha = \alpha_X$ and denote the variance of X_i by σ_i^2 ($=\alpha/w_i$). The joint probability density function (pdf) of $X_1, X_2,..., X_n$ is

$$f(x_1,...,x_n) = \frac{1}{\left(\prod_{i=1}^{n}\sqrt{2\pi}\sigma_i\right)}\exp\left[-\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\sigma_i^2}\right]$$

Our strategy is to change variables in such a way that the independence of *X(mean)* and S_X will be evident. Letting *x(mean)* = $\Sigma w_i x_i$, straightforward computation verifies that

$$\alpha = \frac{1}{\sum_{i=1}^{n} 1/\sigma_i^2}$$

and

$$\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{\sigma_i^2} = \sum_{i=1}^{n}\frac{(x_i-\overline{x})^2}{\sigma_i^2} + (\overline{x}-\mu)/\alpha \tag{5}$$

Hence

$$f(x_1,...,x_n) = \frac{1}{\left(\prod_{i=1}^{n}\sqrt{2\pi}\sigma_i\right)}\exp\left[-\sum_{i=1}^{n}\frac{(x_i-\overline{x})^2}{2\sigma_i^2} - \frac{(\overline{x}-\mu)^2}{2\alpha}\right] \tag{6}$$

Consider the linear transformation $(u_1,...,u_n)=L(x_1,...,x_n)$ defined by u_1 = x(mean), $u_2 = x_2 -$ x(mean),...,$u_n = x_n -$ x(mean), with inverse transformation

$$x_1 = u_1 - \left(\frac{\sigma_1^2}{\sigma_2^2}\right)u_2 - \left(\frac{\sigma_1^2}{\sigma_3^2}\right)u_3 - \ldots - \left(\frac{\sigma_1^2}{\sigma_n^2}\right)u_n,$$
$$x_2 = u_1 + u_2,$$
$$\ldots$$
$$x_n = u_1 + u_n$$

Likewise define new random variables U_1 = *X(mean)*, $U_2 = X_2 -$ *X(mean)*,..., $U_n = X_n -$ *X(mean)*.

If J denotes the Jacobian of L, then the joint pdf of $U_1,\ldots, U_n$ is

$$\frac{J}{\left(\prod_{i=1}^{n}\sqrt{2\pi}\sigma_i\right)}\exp\left[-\frac{\left(-\left(\frac{\sigma_1^2}{\sigma_2^2}\right)u_2 - \left(\frac{\sigma_1^2}{\sigma_3^2}\right)u_3 - \ldots - \left(\frac{\sigma_1^2}{\sigma_n^2}\right)u_n\right)^2}{2\sigma_1^2} - \sum_{i=2}^{n}\frac{u_i^2}{2\sigma_i^2} - \frac{(u_1-\mu)^2}{2\alpha}\right]$$

This now factors as a product of the pdf of U_1 and the joint pdf of $U_2,\ldots, U_n$. Hence U_1 = X(mean) is independent of $U_2,\ldots, U_n$, and hence also independent of

$$\alpha\left[\left(-\left(\frac{\sigma_1^2}{\sigma_2^2}\right)U_2 - \left(\frac{\sigma_1^2}{\sigma_3^2}\right)U_3 - \ldots - \left(\frac{\sigma_1^2}{\sigma_n^2}\right)U_n\right)^2 + \sum_{i=2}^{n}\frac{U_i^2}{\sigma_i^2}\right]$$
$$= \alpha\sum_{i=1}^{n}\frac{\left(X_i - \bar{X}\right)^2}{\sigma_i^2} = S_X$$

Lemma 3:

$S_X/\alpha_X \sim \chi^2(n-1)$ and $S_Y/\alpha_Y \sim \chi^2(m-1)$, where $\chi^2(k)$ denotes the chi-squared distribution with k degrees of freedom.

Proof. The proofs for X and Y are similar. Let

$$A = \sum_{1}^{n} \frac{\left(X_i - \mu_X\right)^2}{\sigma_i^2},$$

$$B = \sum_{1}^{n} \frac{\left(X_i - \bar{X}\right)^2}{\sigma_i^2},$$

and

$$C = \frac{\left(\bar{X} - \mu_X\right)^2}{\alpha_X}.$$

Then by equation 5, $A = B + C$. Since $X_i \sim N(\mu_X, \sigma_i^2)$, $A \sim \chi^2(n)$. Similarly $C \sim \chi^2(1)$. This implies that $B = S_X / \alpha_X \sim \chi^2(n-1)$ provided that B and C are independent, which follows from the proof of lemma 2.

Proposition 2

$$T = \frac{\bar{X} - \bar{Y}}{\sqrt{\frac{S_X/\alpha_X + S_Y/\alpha_Y}{n+m-2}}\sqrt{\alpha_X + \alpha_Y}}$$

is a t-statistic with $n + m - 2$ degrees of freedom.

Proof. Let

$$W = \frac{\bar{X} - \bar{Y}}{\sqrt{\alpha_X + \alpha_Y}}$$

and

$$V = S_X / \alpha_X + S_Y / \alpha_Y .$$

By Lemma 2, W and V are independent. From Lemma 1, W is a standard normal random variable. From Lemma 3, $V \sim \chi^2(n + m - 2)$. Hence

$$T = \frac{W}{\sqrt{V/(n+m-2)}}$$

has the required property.

Application to risk modeling

For certain financial risk factor models, the return to a given factor is computed as the weighted average of returns to the individual securities exposed to that factor. For example, a model for bond credit risk may have a Financial factor to which all financial bonds rated AA are exposed. If the return to this factor is defined to be the duration-weighted average of the option adjusted spread (OAS) returns X_i , we would take weights

$$w_i = \frac{D_i}{\sum_{i=1}^{n} D_i}$$

where D_i is the duration of the ith bond. The factor return is then the weighted average

$$\bar{X} = \sum_{i=1}^{n} w_i X_i$$

We may interpret this factor return as the best linear unbiased estimator of the common mean of a set of independent normal distributions from which the individual bond OAS returns are sampled; the distributions are those of Proposition 1.

If, in the course of building the model, the question arises whether two groups of bond OAS returns $X_1, X_2, \ldots, X_n$ and $Y_1, Y_2, \ldots, Y_m$ share the same mean and therefore should be exposed to the same risk factor, we may use the *t*-statistic of equation 3 to examine the question. A large value of this statistic is evidence that the two groups of bonds have different means and therefore should be exposed to separate risk factors.

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The authors thank Tim Backshall, Oren Cheyette, Anton Honikman, and Darren Stovel for insightful discussions and comments on the manuscript, and Tim Tomaich for help with the U.S. dollar credit model. Special thanks to Justine Withers for a fabulous editing job.

Notes

1- Factors are:

1 EUR_FIN_AAA
2 EUR_FIN_AA
3 EUR_FIN_A
4 EUR_SOV_AA
5 EUR_SOV_A
6 EUR_AGENCY_AAA
7 EUR_UTIL_AA
8 EUR_UTIL_A
9 EUR_UTIL_BBB
10 EUR_ INDUST_AA
11 EUR_ INDUST_A
12 EUR_ INDUST_BBB
13 EUR_ PFAND_AAA
14 EUR_ SUPRA_AAA
15 GBP_FIN_AAA
16 GBP_FIN_AA
17 GBP_FIN_A
18 GBP_INDUST_AA
19 GBP_INDUST_A
20 GBP_INDUST_BBB
21 GBP_SOV_AAA
22 GBP_SOV_AA
23 GBP_SUPRA_AAA
24 GBP_AGENCY
25 USD_CANADIAN_AA
26 USD_CANADIAN_A
27 USD_CANADIAN_BBB
28 USD_ENERGY_AA
29 USD_ENERGY_A
30 USD_ENERGY_BBB
31 USD_FINANCIAL_AAA
32 USD_FINANCIAL_AA
33 USD_FINANCIAL_A
34 USD_FINANCIAL_BBB
35 USD_ INDUST_AAA
36 USD_INDUST_AA
37 USD_ INDUST_A
38 USD_INDUST_BBB
39 USD_UTILITY_AA
40 USD_UTILITY_A
41 USD_SUPRANTL_AAA
42 USD_TRANSPORT_AA
43 USD_TRANSPORT_A
44 USD_TRANSPORT_BBB
45 USD_TELE_AA
46 USD_TELE_A
48 USD_TELE_BBB
49 USD_YANKEE_AAA
50 USD_YANKEE_AA
51 USD_YANKEE_A
52 USD_YANKEE_BBB

2- These observations do not extend to bonds that are below investment grade. Empirical evidence shows that in the U.S. market, there is little correlation between below and above investment grade bonds in the same sector.

3- The Utility BBB sector is missing in the remapped model since virtually all the EuroBIG BBB Utility bonds were in the Telecommunications subsector.

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TABLES & FIGURES

Table 1: The number of bonds available in each sector-by-rating bucket on May 31, 2001.

		AAA	AA	A	BBB
Euro	Agency	96	24		
	Financial	44	56	33	
	Industrial		9	46	17
	Sovereign		4	8	5
	Supranational	26			
	Utility		10	32	11
	Pfandbrief	184			
Sterling	Financial	17	36	13	
	Industrial		7	55	19
	Sovereign	14	12		
	Supranational	32			
U.S. Dollar	Canadian		18	66	61
	Supranational	26			
	Transportation		7	15	72
	Utility		7	91	127
	Energy		9	17	134
	Telecommunications		55	76	33
	Industrial	13	57	310	375
	Yankee	11	28	79	124
	Financial	29	168	527	147
	Agency	268			

Tables 2a–d: Financial sector correlations within and across markets as of May 31, 2001. Most within-market correlations are close to one and cross-market correlations are closer to zero. Correlations between sterling and U.S. dollar sectors tend to be stronger than correlations between euro sectors and sectors in other markets.

Table 2a: Correlations between euro Financial sectors.

	FIN_AAA	**FIN_AA**	**FIN_A**
FIN_AAA	1.00	0.850	0.76
FIN_AA		1.00	0.86
FIN_A			1.00

Table 2b: Correlations between sterling Financial sectors.

	FIN_AAA	**FIN_AA**	**FIN_A**
FIN_AAA	1.00	0.61	0.65
FIN_AA		1.00	0.96
FIN_A			1.00

Table 2c: Correlations between U.S. dollar Financial sectors.

	FIN_AAA	**FIN_AA**	**FIN_A**	**FIN_BBB**
FIN_AAA	1.00	0.77	0.66	0.73
FIN_AA		1.00	0.94	0.91
FIN_A			1.00	0.91
FIN_BBB				1.00

Table 2d: Cross-market Financial sector correlations.

	FIN_AAA	FIN_AA	FIN_A
Euro/sterling	0.10	–0.18	–0.06
Euro/U.S. dollar	0.11	–0.06	–0.05
Sterling/U.S. dollar	0.12	0.44	0.51

Tables 3a–d: Correlations within markets and across markets for AA factors. May 31, 2001.

Table 3a: Correlations between euro AA sectors.

	FIN_AA	SOV_AA	UTIL_AA	INDUST_AA
FIN_AA	1.00	0.82	0.56	0.73
SOV_AA		1.00	0.27	0.67
UTIL_AA			1.00	0.61
INDUST_AA				1.00

Table 3b: Correlations between sterling AA sectors.

	FIN_AA	INDUST_AA
FIN_AA	1.00	0.87
INDUST_AA		1.00

Table 3c: Correlations between U.S. dollar AA sectors.

	CANADIAN_AA	ENERGY_AA	FIN_AA	INDUST_AA	UTIL_AA	TRANSPORT_AA	TELE_AA	YANKEE_AA
CANADIAN_AA	1.00	0.52	0.67	0.80	0.75	0.47	0.82	0.59
ENERGY_AA		1.00	0.41	0.63	0.80	0.80	0.74	0.55
FIN_AA			1.00	0387	0.60	0.45	0.75	0.79
INDUST_AA				1.00	0.79	0.57	0.86	0.72
UTIL_AA					1.00	0.71	0.94	0.60
TRANSPORT_AA						1.00	0.63	0.64
TELE_AA							1.00	0.72
YANKEE_AA								1.00

Table 3d: Cross-market correlations between AA sectors.

	FIN_AA	INDUST_AA	UTILITY_AA
Euro/sterling	–0.18	–0.12	
Euro/U.S. dollar	–0.06	–0.02	–0.05
Sterling/U.S. dollar	0.44	0.06	

Table 4: Relatively high correlations between euro Utilities and sterling Industrials, near zero correlations between euro Industrials and sterling Industrials. May 31, 2001.

		Sterling		
		INDUST_AA	INDUST_A	INDUST_BBB
Euro	UTIL_AA	0.35	0.33	0.52
	UTIL_A	0.32	0.36	0.47
	UTIL_BBB	0.47	0.51	0.56
	INDUST_AA	–0.12	–0.06	0.02
	INDUST_A	0.12	0.12	0.23
	INDUST_BBB	–0.22	–0.25	–0.07

Table 5: Telecommunications bonds in EuroBIG remapped to the industrial sector. This remapping has diminished anomalous correlations between euro Utilities and sterling Industrials and slightly increased correlations between the euro and sterling Industrial sectors. May 31, 2001.

			Sterling	
		INDUST_AA	INDUST_A	INDUST_BBB
Euro	UTIL_AA	0.20	0.33	0.29
	UTIL_A	0.21	0.26	0.31
	INDUST_AA	0,17	0.18	0.38
	INDUST_A	0.26	0.27	0.39
	INDUST_BBB	0.23	0.25	0.37

Table 6: Examples of bonds issued by the same entity but on different markets.

Issuer		**Euro Bond**		**Sterling Bond**		**U.S. Dollar Bond**	
Name	**Sector**	**Maturity**	**Coupon (%)**	**Maturity**	**Coupon (%)**	**Maturity**	**Coupon (%)**
Government of Canada	SOV/CAN	2008/07/07	4.875	2004/11/26	6.25	2002/07/15	6.125
Dresdner Bank	FIN	2005/05/25	5.0	2007/12/07	7.75	2005/09/15	6.625
European Investment Bank	SUPRA	2007/02/15	5.75	2003/06/10	8.0	2002/06/01	9.125
Toyota	INDUST	2003/11/10	4.75	2007/12/07	6.25	2003/11/13	5.625

Table 7: Examples of cross-market correlations for individual issuers. Correlations were computed using the bonds given in Table 6.

Issuer	Euro/Sterling Correlation	Euro/U.S. Dollar Correlation	Sterling/U.S. Dollar Correlation
Government of Canada	0.12	–0.10	0.07
Dresdner Bank	–0.51	–0.01	–0.03
European Investment Bank	0.17	–0.08	0.01
Toyota	–0.21	–0.19	0.23

Table 8a: Euro market

	INDUST_AA	INDUST_A	INDUST_BBB
INDUST_AA	1.00	0.60	0.41
INDUST_A		1.00	0.67
INDUST_BBB			1.00

Table 8b: Sterling market

	INDUST_AA	INDUST_A	INDUST_BBB
INDUST_AA	1.00	0.88	0.88
INDUST_A		1.00	0.82
INDUST_BBB			1.00

Table 8c: U.S. dollar market

	INDUST_AAA	INDUST_AA	INDUST_A	INDUST_BBB
INDUST_AAA	1.00	0.87	0.78	0.78
INDUST_AA		1.00	0.91	0.87
INDUST_A			1.00	0.96
INDUST_BBB				1.00

Table 8d: Across markets

	INDUST_AA	INDUST_A	INDUST_BBB
Euro/Sterling	–0.12	0.12	–0.07
Euro/U.S. Dollar	–0.02	0.13	0.21
Sterling/U.S. Dollar	0.06	0.19	0.17

Table 9: Supranational AAA bonds

	Euro	Sterling	U.S. Dollar
Euro	1.00	0.02	–0.06
Sterling		1.00	0.16
U.S. Dollar			1.00

Figure 1: Cross-market volatility comparison. U.S. dollar factor return volatilities generally exceed euro and sterling by a factor of two to three. Estimates are as of May 31, 2001, based on monthly data weighted exponentially with a 24-month half-life.

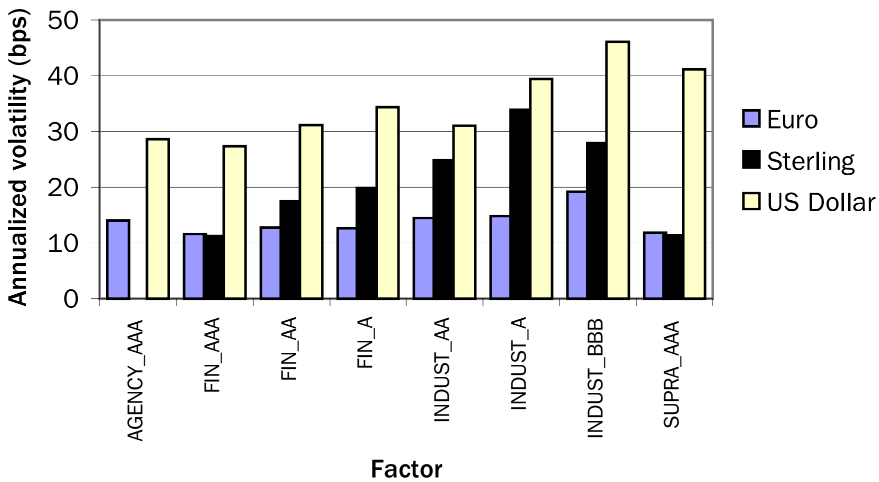


Figure 2: Color-coded map of spread return correlations for the euro, sterling, and U.S. dollar markets. High correlations (0.7–1.0) are consistently observed within a single market, whereas cross-market correlations remain mostly between –0.3 and 0.3. On average, the correlation matrix shows a clear cross-market de-correlation[1].

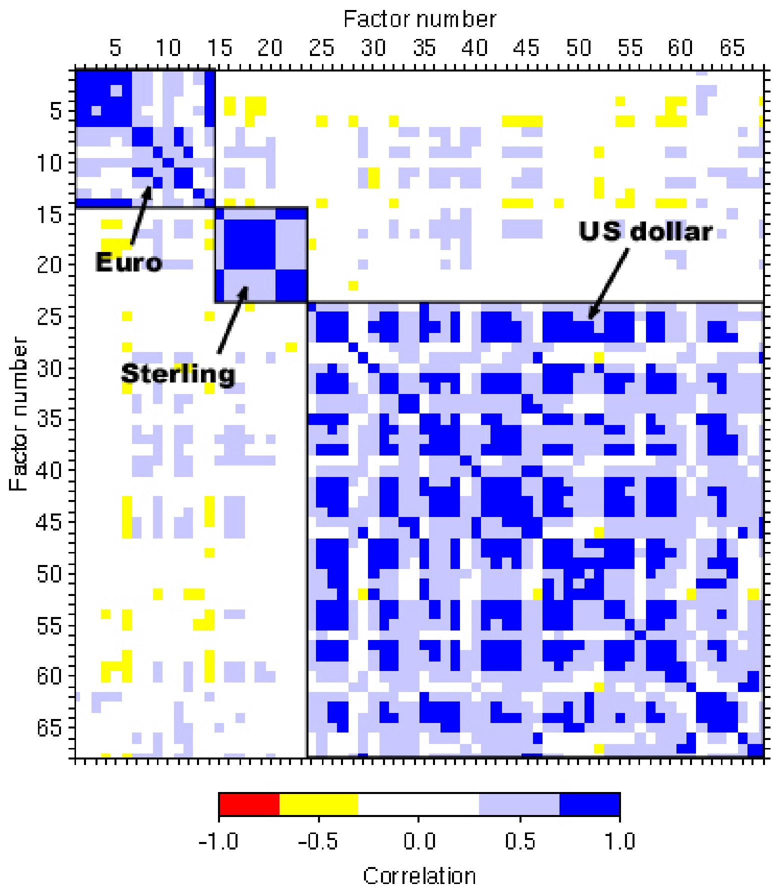
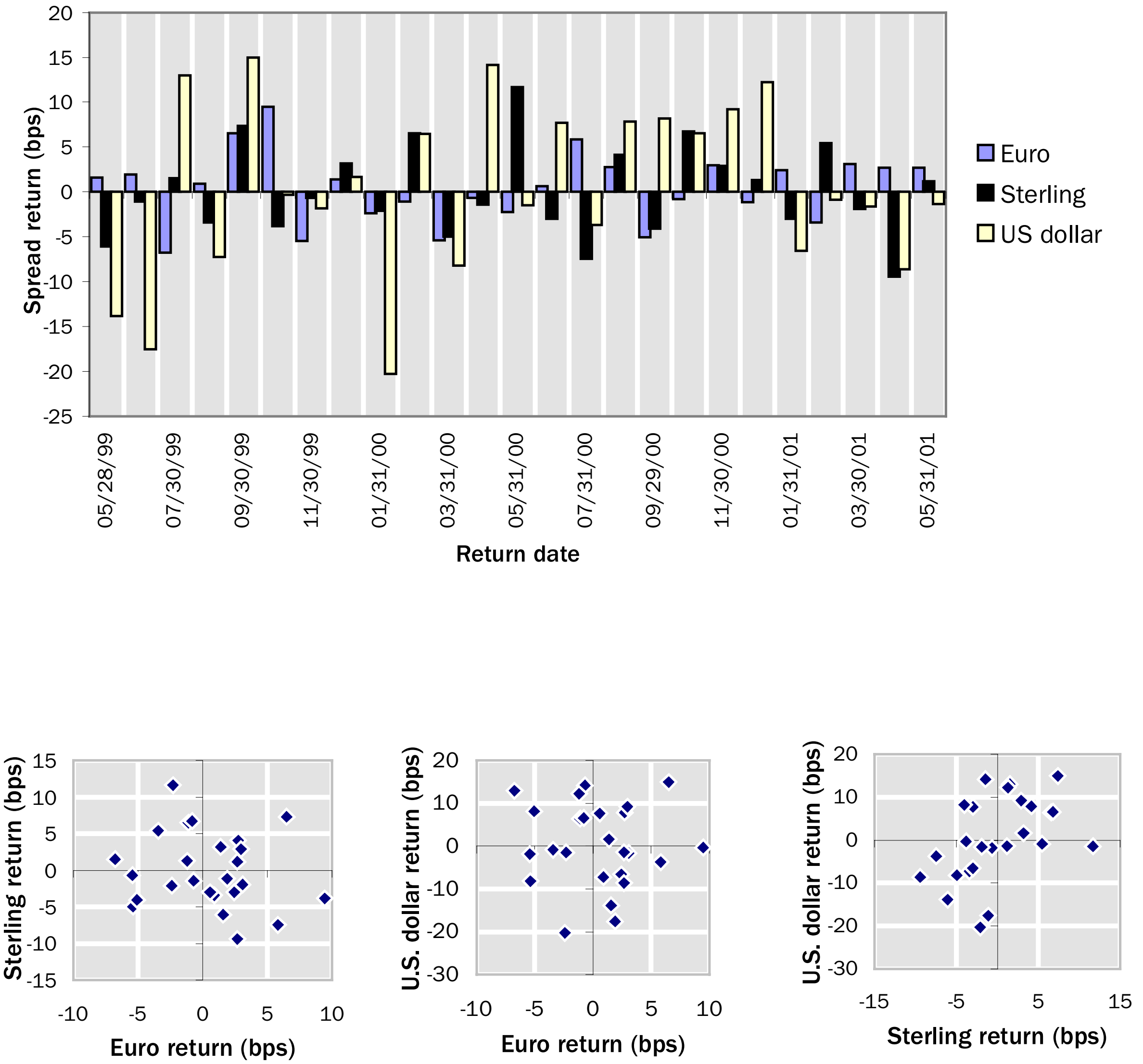


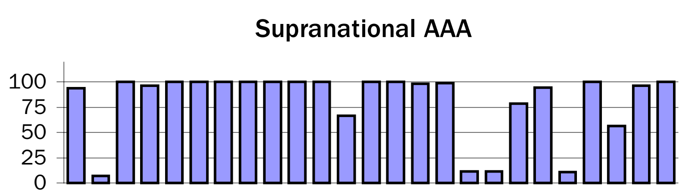
Figure 3: Cross-market comparison of Financial AA spread factor returns. Top: return time series. Bottom, left to right: sterling returns plotted as a function of euro returns, and U.S. dollar returns plotted as a function of euro and sterling returns. The corresponding correlations are –0.15, –0.05, and +0.45 respectively.

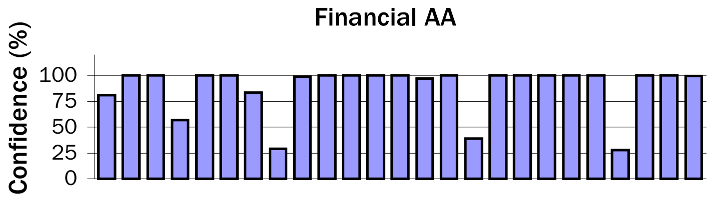


Figures 4a–b: Confidence levels of rejection of the null hypothesis that the mean returns to a credit factor are the same across currencies. In most cases, returns are shown to have different means. Top: Supranational AAA. Middle: Financial AA. Bottom: Industrial A.

Figure 4a: Euro versus sterling

Figure 4b: Euro versus U.S. dollar




Financial AA
Confidence (%)
100
75
50
25
0

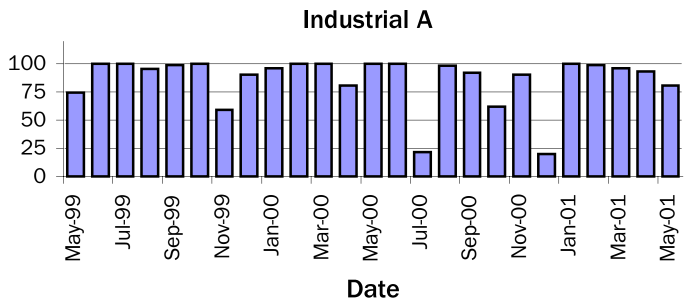

Industrial A
100
75
50
25
0
May-99
Jul-99
Sep-99
Nov-99
Jan-00
Mar-00
May-00
Jul-00
Sep-00
Nov-00
Jan-01
Mar-01
May-01
Date

Figure 5: Cross-market comparison of spread returns for three bonds issued on different markets by Toyota. The cross-market correlations corresponding to the bottom panels are, from left to right, –0.21, –0.19, and 0.23.

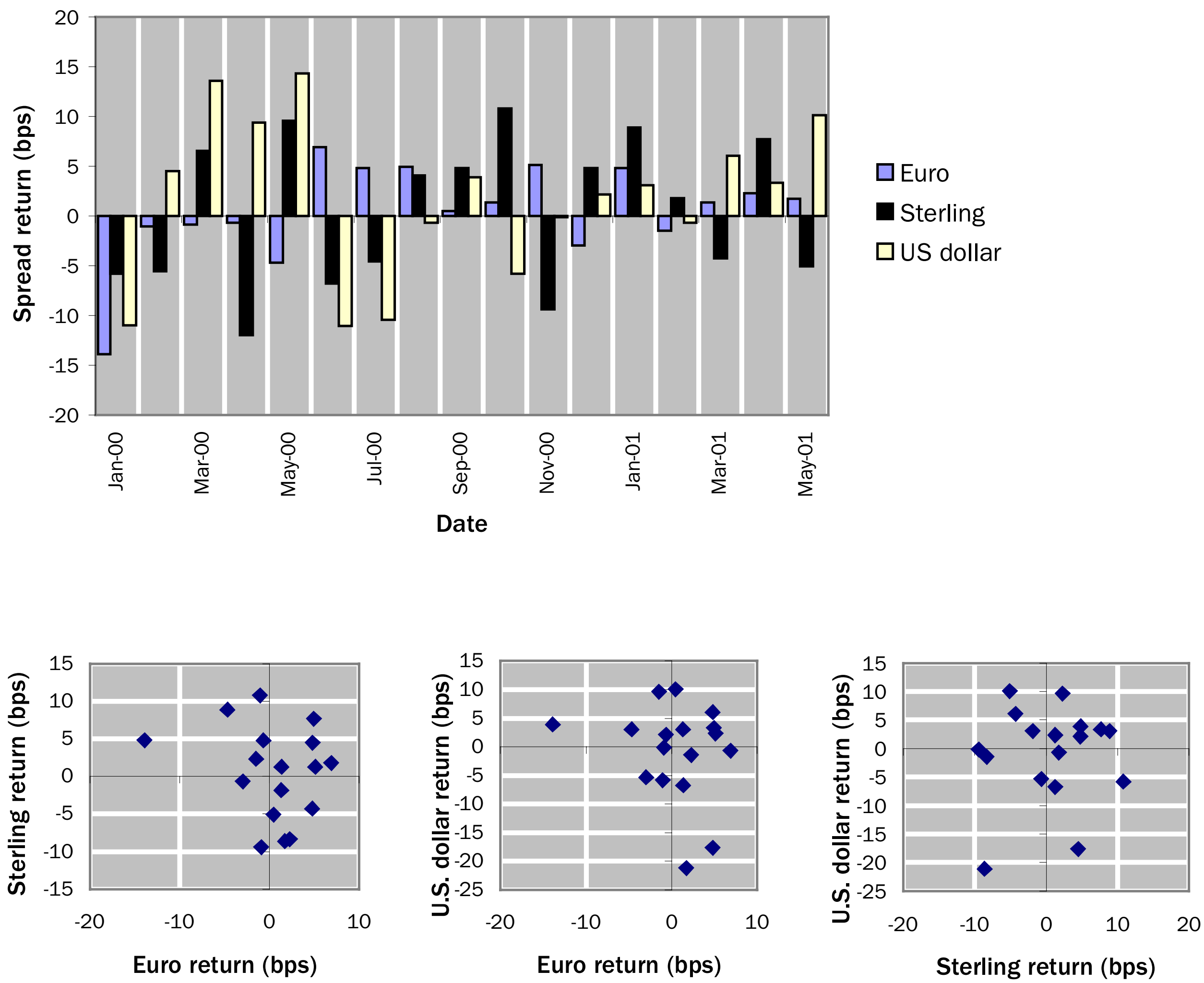


Figure 6: Spread returns of bonds issued by the European Investment Bank on the euro, sterling, and U.S. dollar markets plotted as a function of amount outstanding, maturity, duration, and coupon. There is no clear relation between either of these factors and the returns.

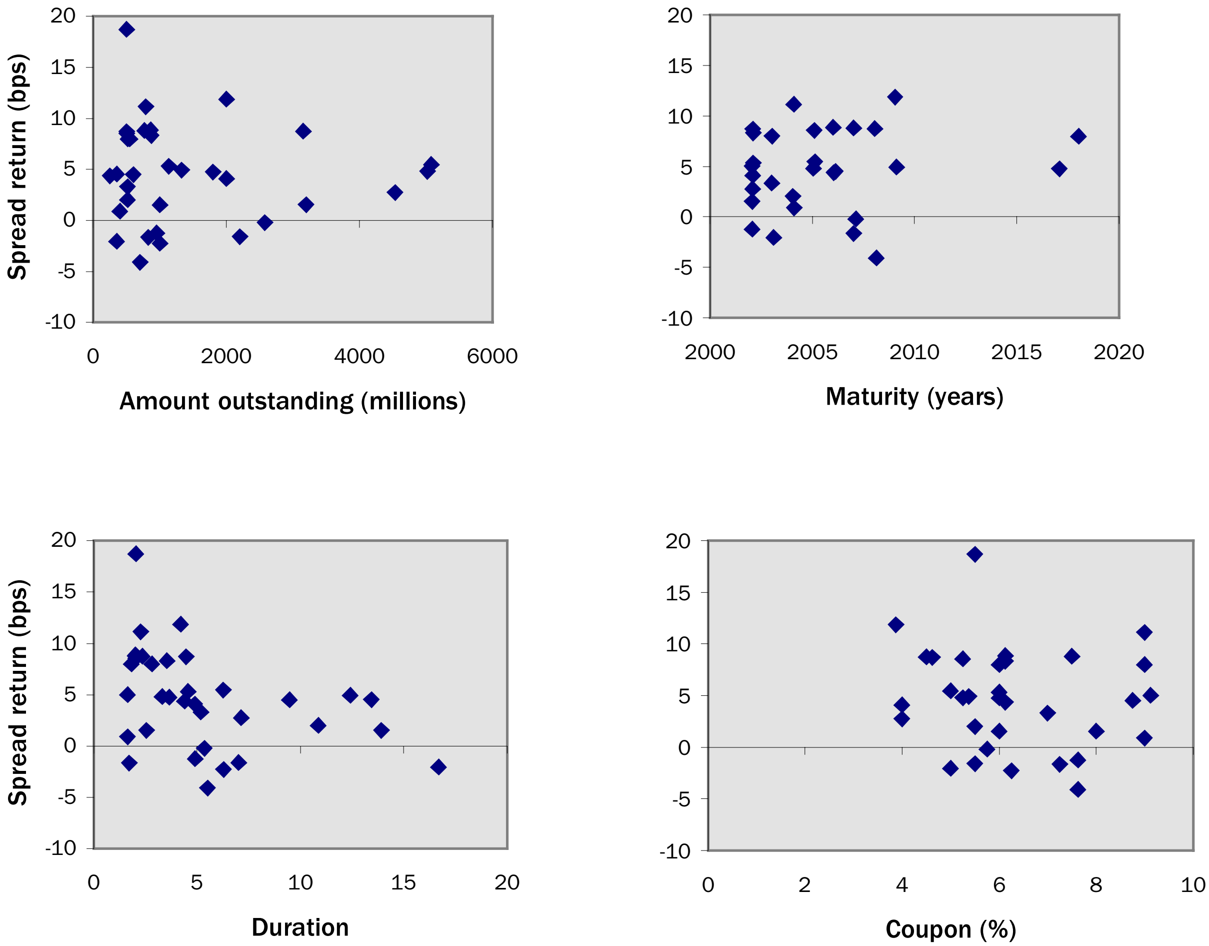


Figure 7: Cross-market comparison of Supranational AAA spread factor returns. Top: return time series. Bottom, from left to right: sterling returns plotted as a function of euro returns, and U.S. dollars returns plotted as a function of euro and sterling returns. The corresponding correlations are 0.026, -0.083, and 0.23 respectively. Supranational AAA spread returns appear to be relatively independent across the euro, sterling and U.S. dollar markets.

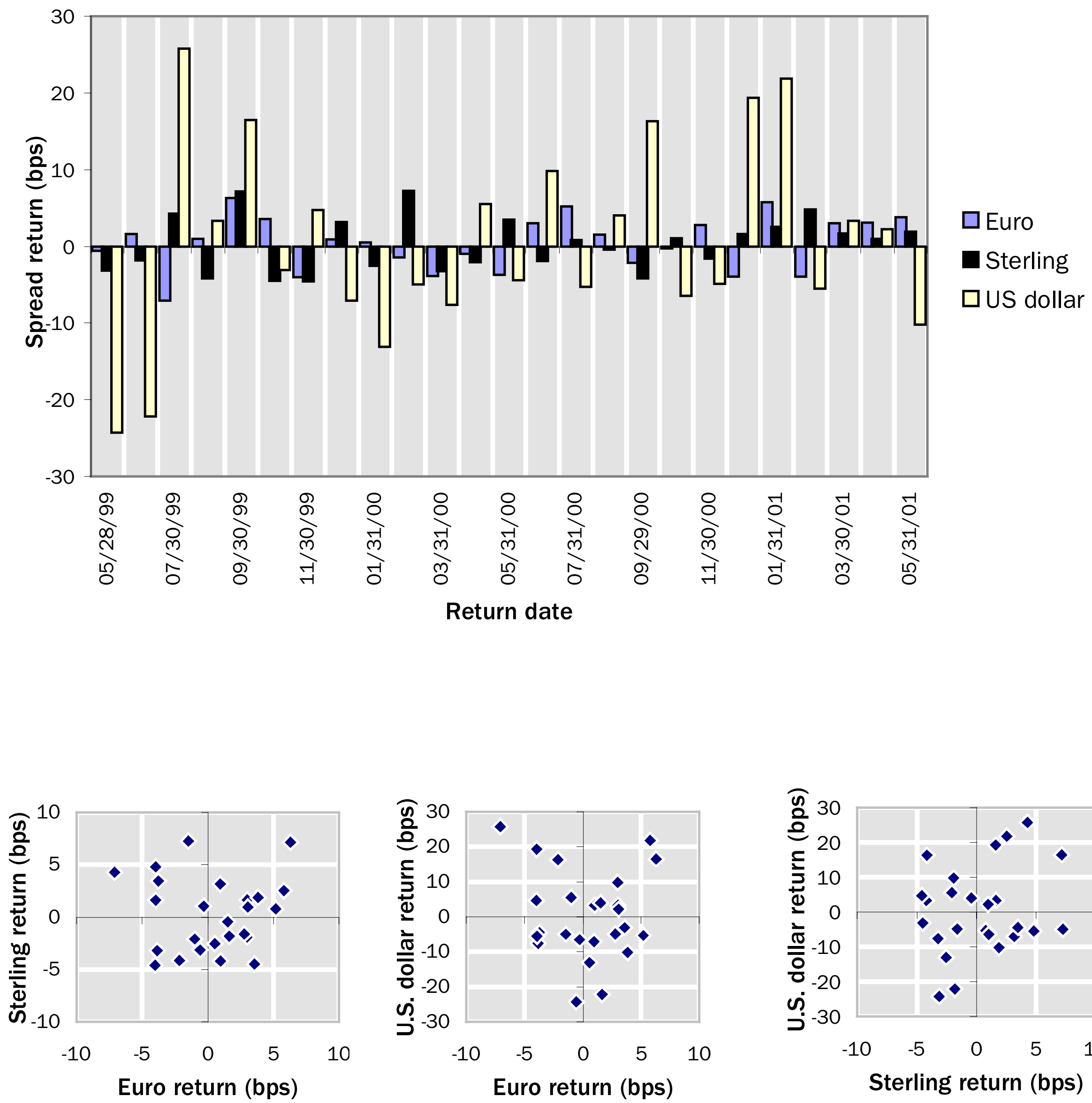


Figure 8: Cross-market comparison of Industrial A spread factor returns. Top: return time series. Bottom, from left to right: sterling returns plotted as a function of euro returns, and U.S. dollar returns plotted as a function of euro and sterling returns. The corresponding correlations are 0.12, 0.17, and 0.20 respectively. Note, again, the independence of spread returns across markets.

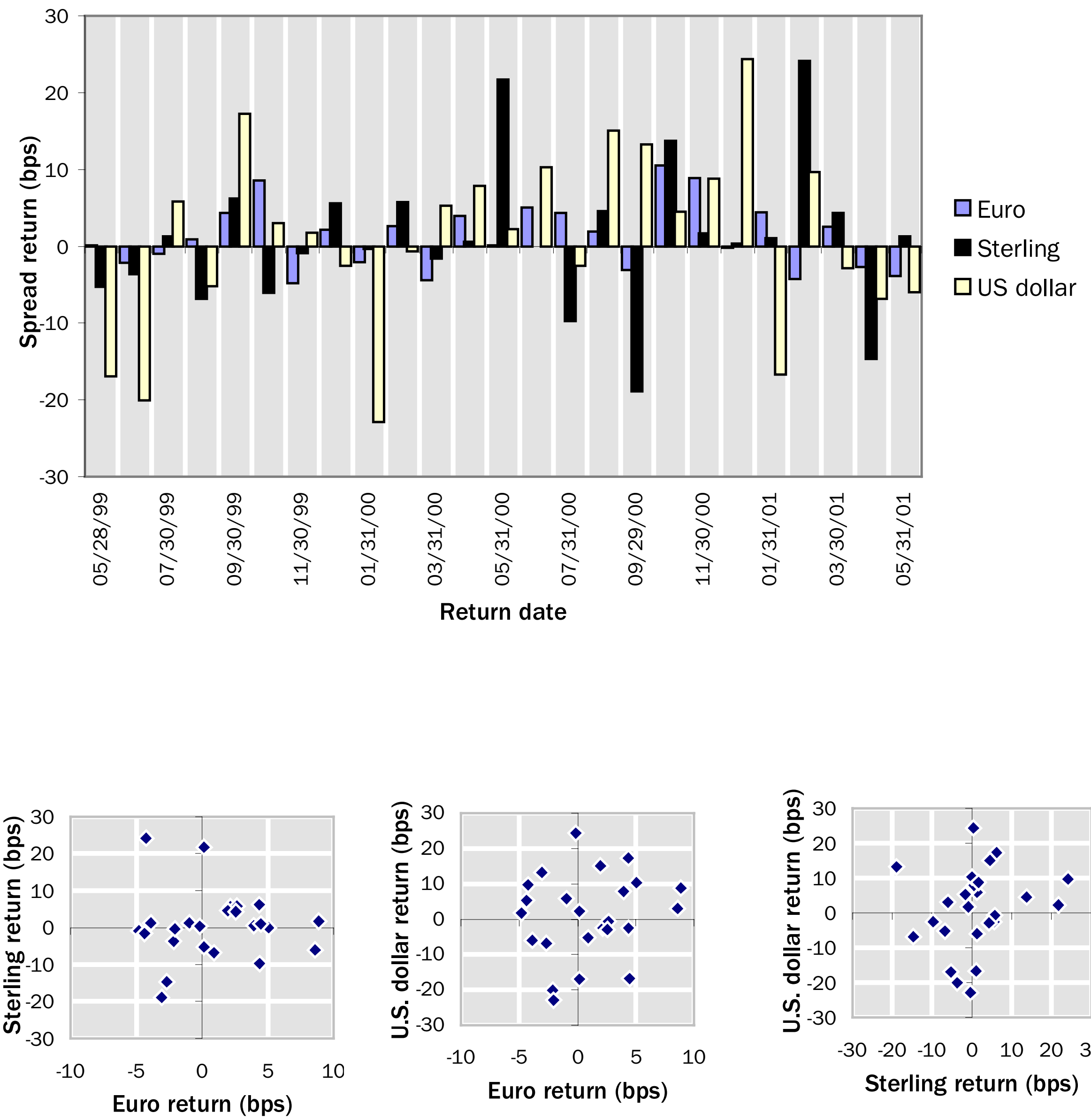


Figure 9a: Euro AA credit spread returns

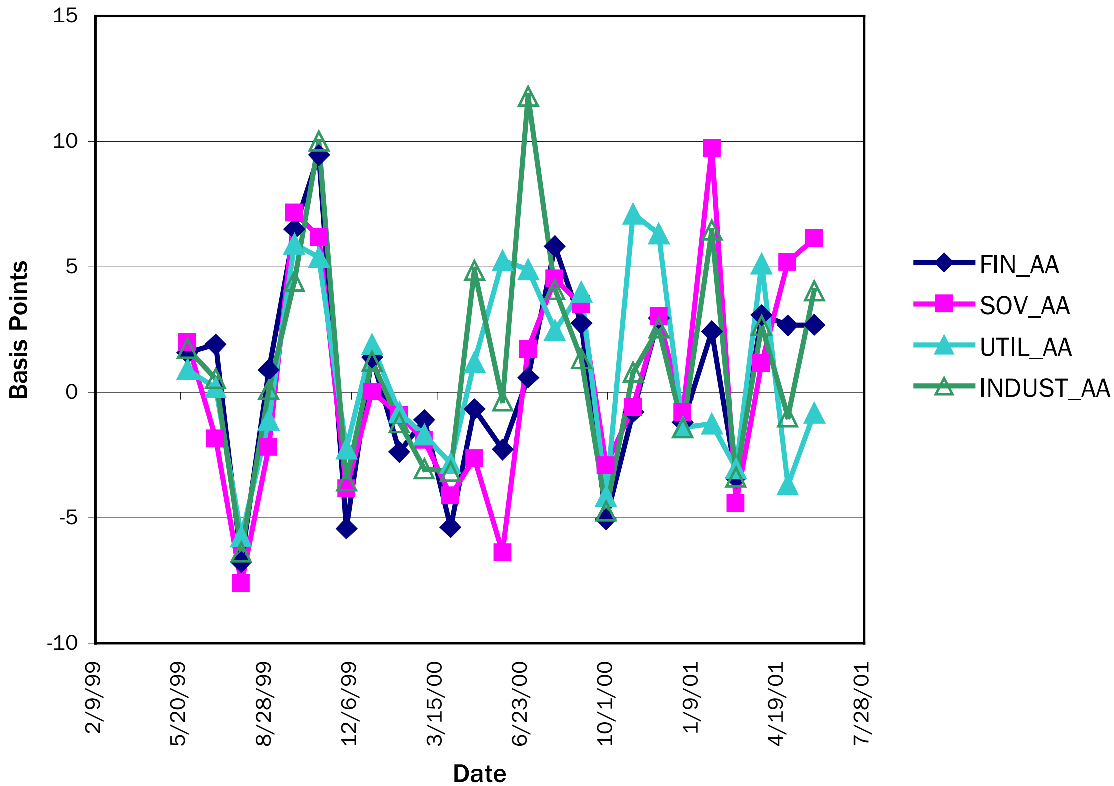


Figure 9b: U.S. dollar AA credit spread returns

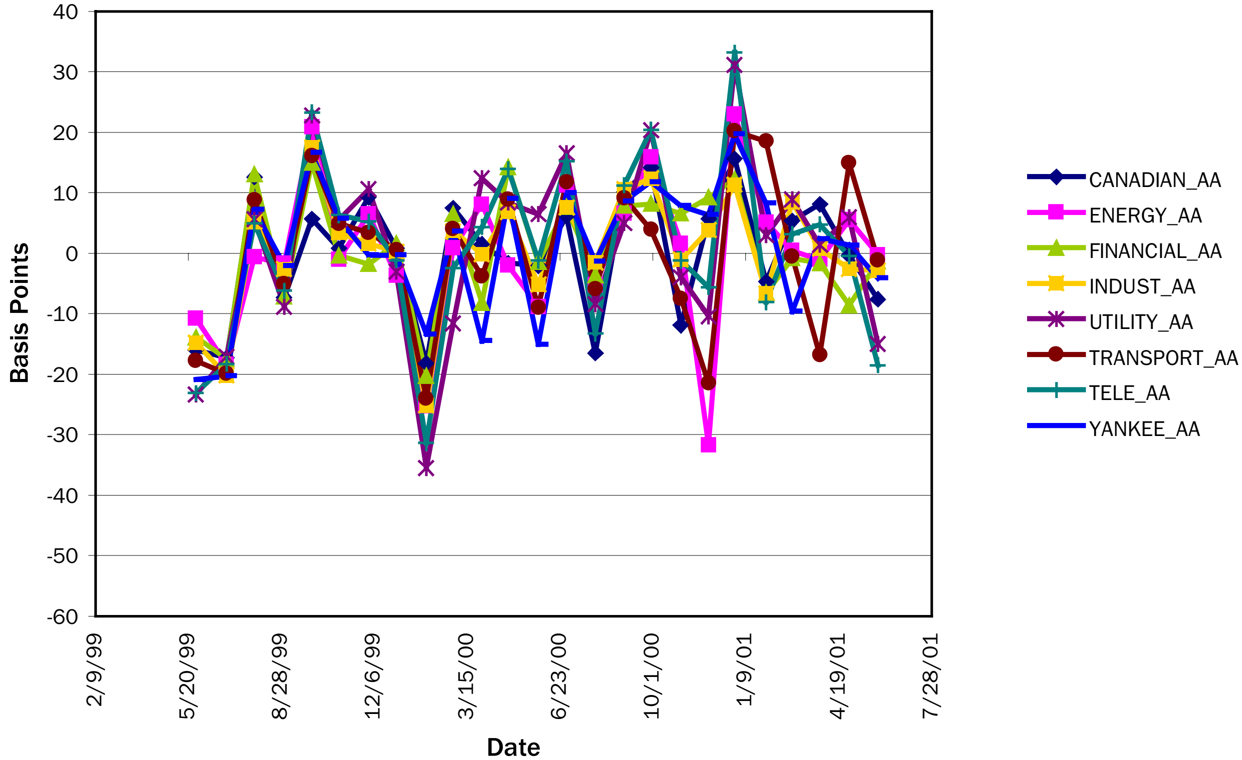


Figure 9c: Sterling AA credit spread returns