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Visualization and Mathematics III

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Preface

Mathematical Visualization aims at an abstract framework for fundamental objects appearing in visualization and at the application of the manifold visualization techniques to problems in geometry, topology and numerical mathematics. The articles in this volume report on new research results in this field, on the development of software and educational material and on mathematical applications.

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The themes of this book cover important recent developments in the following fields:

- Geometry and Combinatorics of Meshes
- Discrete Vector Fields and Topology
- Geometric Modelling
- Image Based Visualization
- Software Environments and Applications
- Education and Communication

We hope that the research articles of this book will stimulate the readers' own work and will further strengthen the development of the field of Mathematical Visualization.

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