Comparison of Variations in the Marching Cubes Algorithm for Cortical Surface Reconstructions

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INTRODUCTION

Cortical surface reconstructions are increasingly being used to study the anatomy and functional processing of the brain and to make comparisons across individuals. Although the geometry of the surface of the brain varies across individuals, its topology is always equivalent to a two-sphere with an Euler Characteristic(χ) of 2. Many different algorithms exist for reconstructing cortical surfaces. One of the most common algorithms used for constructing a surface from volume data is the Marching Cubes algorithm[1], namely because of its speed. We are investigating how variations in this algorithm compare geometric and topological properties in the resulting cortical surface. To our knowledge, this is the first comparison of geometric and topological cortical surface properties to be performed with the Marching Cubes algorithm.

The original Marching Cubes algorithm developed by Lorensen and Cline can lead to many topological problems in the reconstructed surface; as a result, many modifications to the algorithm have been proposed to correct or reduce these defects. The defects we will be discussing include holes, non-manifold edges and vertices, and handles. We investigate the use of several modifications to the original Marching Cubes Algorithm including the effect of lookup tables, isovalue selection, and data perturbation when reconstructing cortical surfaces.

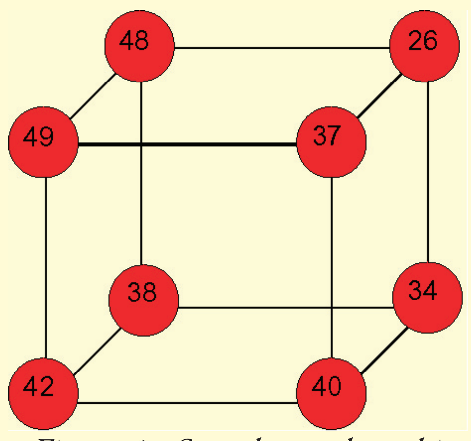


Figure 1: Sample voxel used in Marching Cubes Algorithm

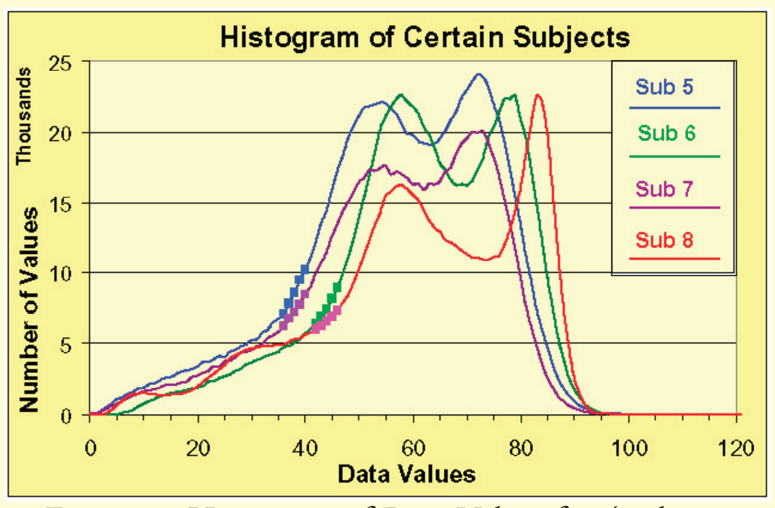


Figure 2: Histogram of Data Values for 4 subjects. The range of isovalues chosen for each subject is shown by points in a lighter color.

METHODS

This study utilized 11 T-1 weighted anatomical MRI scans of subjects from a static force experiment[2]. Each scan set is comprised of 256 x 256 x180 voxels with dimensions .86 mm x .86 mm x 1.0 mm. The scalp, skull, cerebellum, and other non-brain tissue were stripped and the cerebrum was divided into left and right hemispheres. Inhomogeneities were corrected using the MNI N3 algorithm[2] before the Marching Cubes algorithm was applied. We implemented novel modifications to the Marching Cubes algorithm including 3 newly created lookup tables and a data perturbation method for use with integer isovalues. We applied these modified algorithms to the left hemisphere of each subject, using 5 integer and 5 non-integer isovalues and 4 lookup tables per isovalue, leading to 40 generated surfaces per subject.

A Marching Cubes algorithm uses a set of data values with an underlying cubical structure, such as MRI or CT data. The cubes, or voxels, used in the algorithm are rectangular prisms that have a data value at each corner (Fig. 1). A surface is constructed by selecting a constant value, known as the isovalue, and assembling a level set corresponding to this isovalue. This is done by visiting each voxel, using a lookup table to identify triangles in this voxel belonging to our surface, and constructing the surface from all of the identified triangles.

Isovalue selection is a critical decision that influences many surface characteristics such as the Euler characteristic (χ=V-E+F, where V is number of vertices, E is number of edges, and F is number of faces) and surface area. We use a histogram of the MRI data values to choose a reasonable isovalue. Typically, an isovalue near the base of a histogram peak is selected in order to generate a surface. Figure 2 shows the histograms of several subjects and the range of isovalues chosen to generate a surface. Other methods can also be used to select an appropriate isovalue[3][4]. We chose several values to demonstrate its effect on the resulting surface.

Once an isovalue has been chosen, the Marching Cubes algorithm may be used to construct our surface. We identify a given voxel according to the values it contains on its corners. Each edge of the voxel that terminates in a value higher than the isovalue (large dots in figures 3-4) and a value lower than the isovalue (no dots in figures 3-4) will have a point along the edge that is equal to the isovalue (small dots in figures 3-4). These edge points belong in our surface and will be referred to as surface points. Once surface points from a given voxel have been identified, we must determine how to connect these points with edges and triangular faces. This is done through the use of a lookup table, and is the source of much of the variability in the Marching Cubes algorithm.

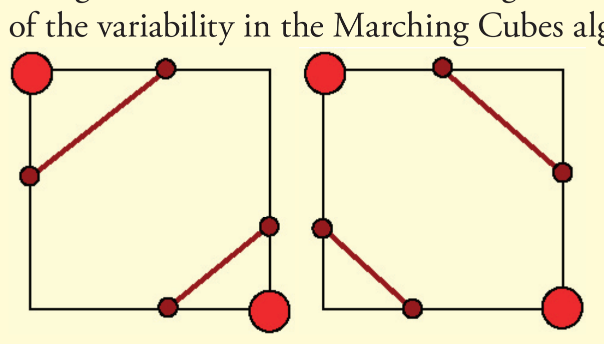


Figure 3: Two ways to connect an ambiguous face.

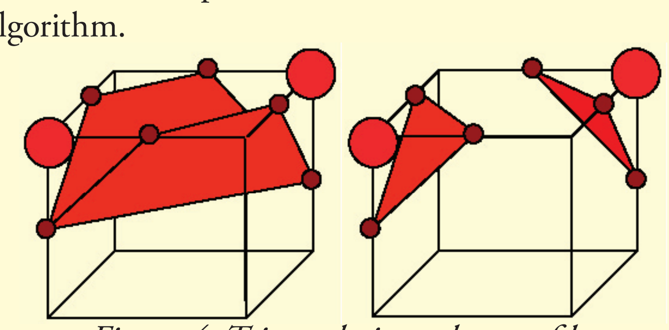


Figure 4: Triangulation scheme of low and tetra table (left) and high table (right). Asymptotic choice dependent on exact values.

The significant variability in lookup tables is a direct result of how surface points are connected in voxels that have an ambiguous face. An ambiguous face is a face of our voxel with two values higher than the isovalue on opposite corners and two values lower than the isovalue on opposite corners, yielding two valid ways that edges may connect the surface points on this face (Fig. 3). The lookup table defines which connectivity scheme is used in each voxel case. The original lookup table proposed by Lorensen and Cline was defined inconsistently for cubes sharing an ambiguous face, so that edges on ambiguous faces did not match up, resulting in holes in the final surface. Thus, it is important to consistently define what is done with ambiguous faces when creating a lookup table.

The 4 lookup tables that we implemented will be referred to as Low, High, Tetra, and Asymptotic. The Low table always separates the low data values of an ambiguous face. The High table always separates the high values of an ambiguous face. The Tetra table decides how to connect an ambiguous face based upon a tetrahedral decomposition of the cube. The Asymptotic table uses the asymptotic decider[5] to decide how to connect an ambiguous face. A sample cube and how each table would triangulate it is shown in Figure 4. It is important to note that the topological variability that comes from using these different connectivity schemes is caused by the cubes with ambiguous faces.

Another issue encountered in a Marching Cubes type algorithm is what to do when a data value equals the isovalue. The original algorithm simply classifies these data points as lower than the isovalue, and places the surface point in the same position as the data value. This can lead to non-manifold vertices and edges in the resulting surface that prevent us from using our surface in other applications such as handle removal and flattening. There are two ways to prevent these defects from occurring in a surface. Choosing an isovalue that is not an integer prevents any data values from being equal to our isovalue, resulting in an elimination of any non-manifold defects. We may also prevent non-manifold defects by perturbing the data values that are equal to the isovalue so that they become only one value higher or lower than the isovalue.

Choosing to perturb data values forces more decisions regarding the direction of perturbation. Surfaces resulting from always perturbing values down or always perturbing values up will have different properties. We created and implemented a perturbation method for our algorithm that decides the direction of perturbation based upon the neighboring data values. We take all 26 data values that are part of the 8 cubes our candidate data value belongs to and determine if the majority of them are higher or lower than the isovalue. The candidate value is then perturbed in the direction of the majority of its neighbors.

The surface properties examined after using the various algorithms include surface area, number of triangles, maximum vertex degree, and Euler characteristic. The maximum vertex degree is the largest number of triangles that meet at a vertex in our surface. The Euler characteristic is a measure of the number of handles in the surface with more handles increasing the value in the negative direction.

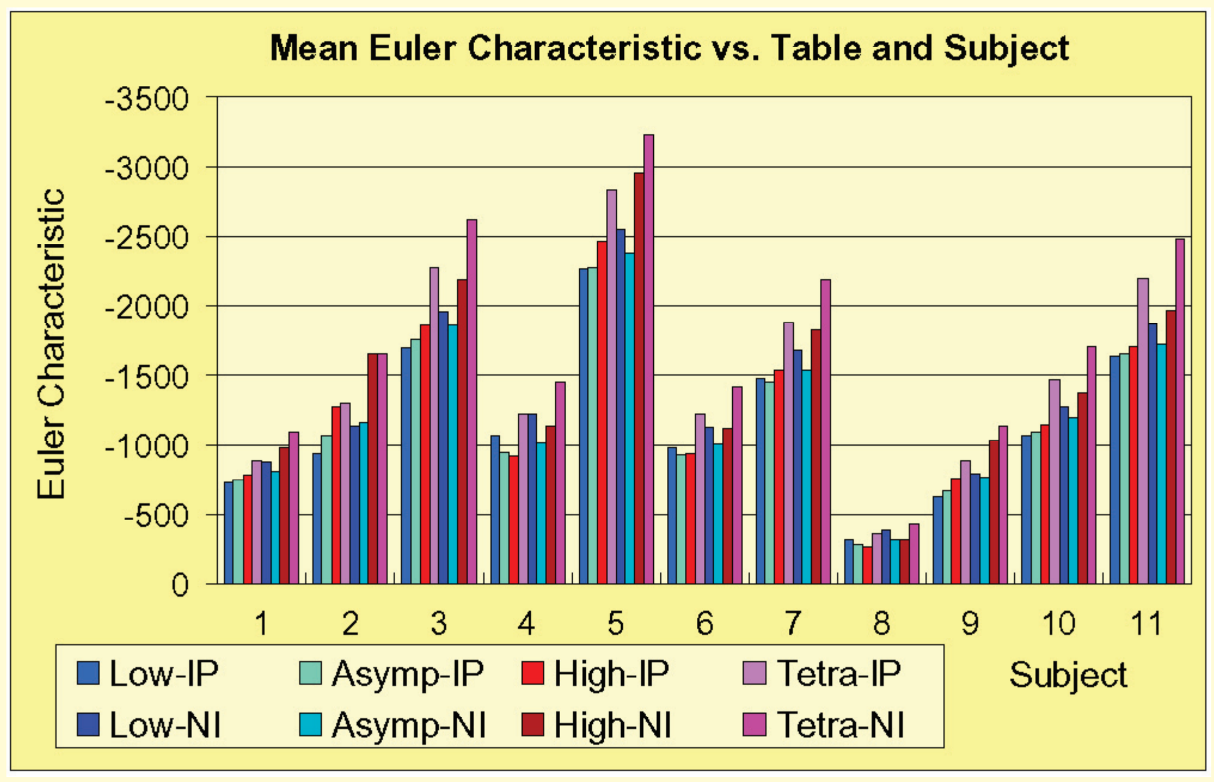


Figure 5: Mean χ values for each table using integer isovalues with perturbing (IP) and non-integer isovalues (NI). Means were taken over the five isovalues tested for each subject and method.

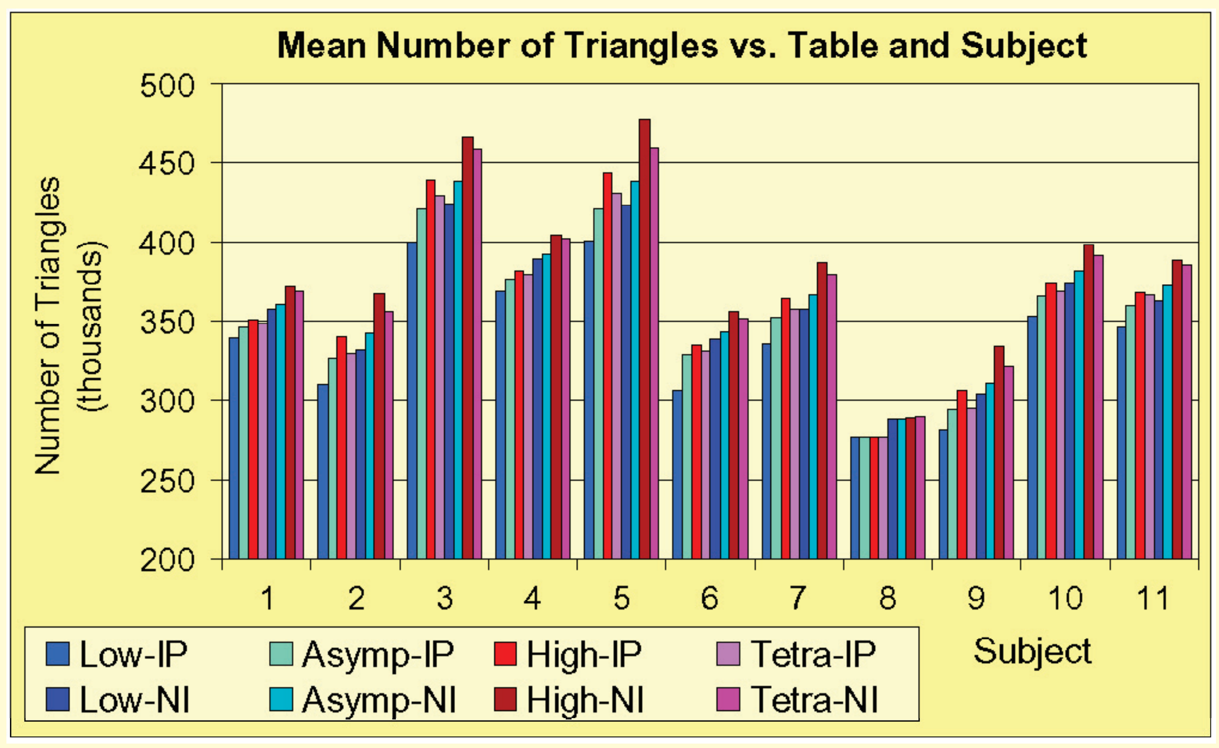


Figure 6: Mean Number of Triangles for each table using integer isovalues with perturbing (IP) and non-integer isovalues (NI). Means were taken over the five isovalues tested for each subject and method.

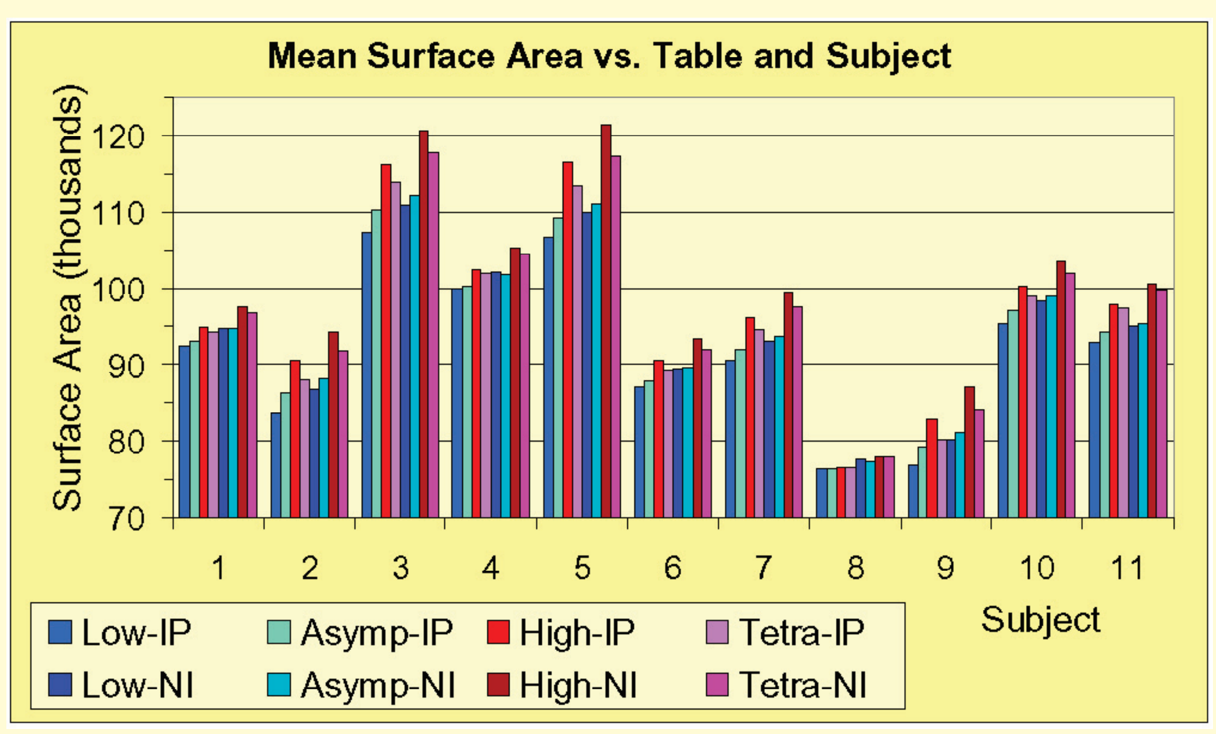


Figure 7: Mean Surface Area values for each table using integer isovalues with perturbing (IP) and non-integer isovalues (NI). Means were taken over the five isovalues tested for each subject and method.

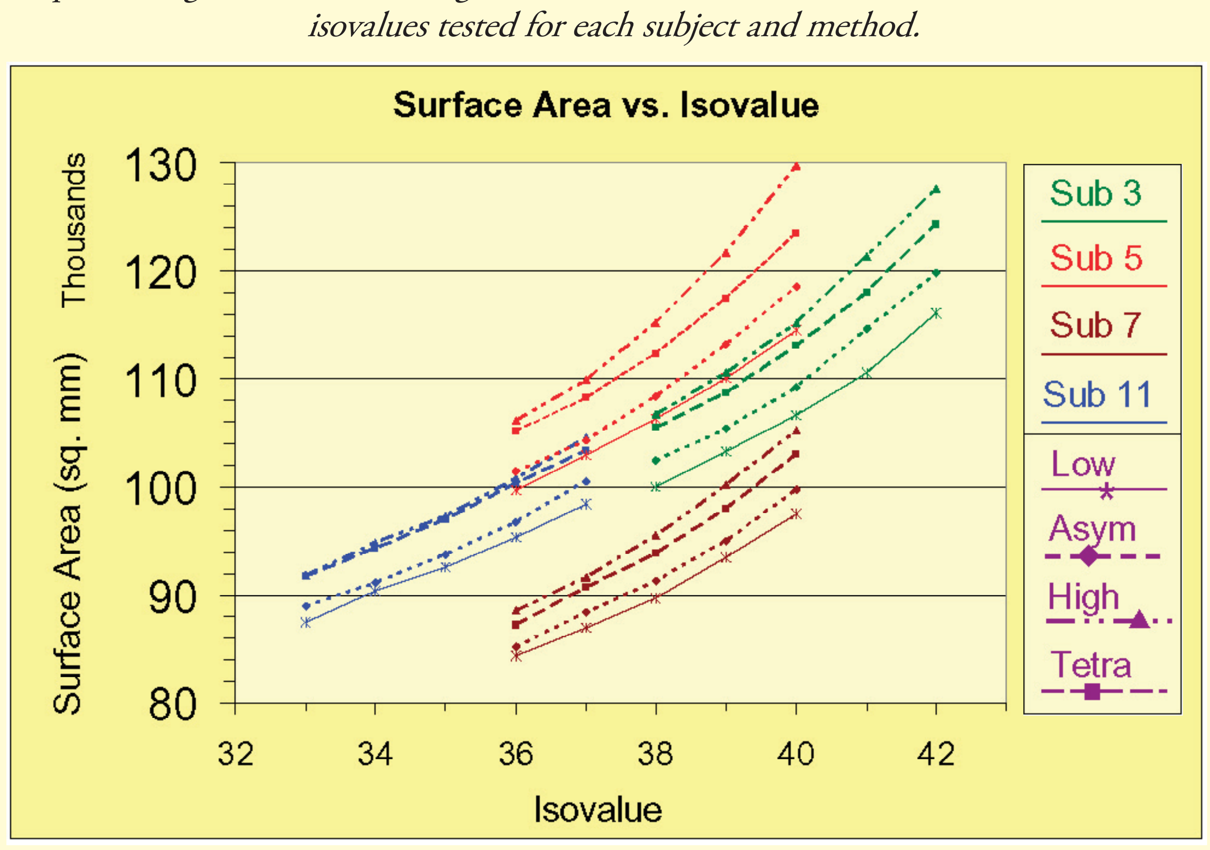


Figure 8: Surface Area values for the five isovalues tested in 4 different subjects.

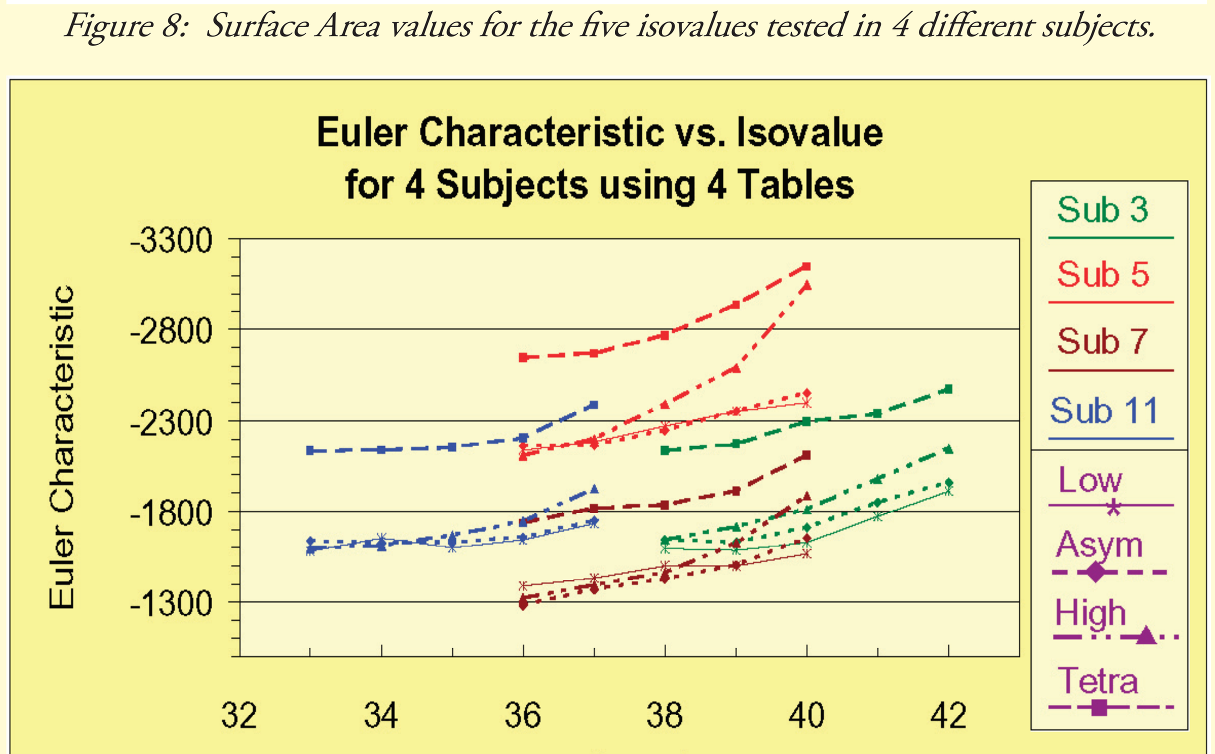
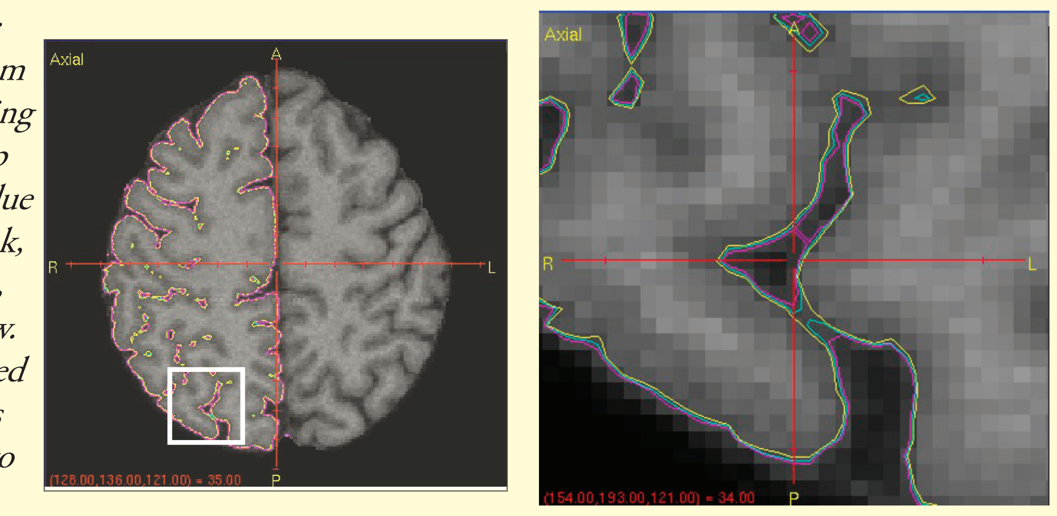


Figure 9: Euler characteristic values for the five isovalues tested in 4 different subjects.

RESULTS

We have employed several modifications to the Marching Cubes algorithm, resulting in a variety of method combinations that could be used to generate a surface. We wish to discern which algorithm can reliably give us the most desirable surface characteristics such as χ as close as possible to 2, small number of triangles, and low vertex degree. A small number of triangles and vertex degree are desirable features since the time to run many downstream applications is dependent upon them. Figures 5-6 demonstrate that choosing to perturb data values that are equal to the isovalue results in more desirable surface properties than using the same table with a non-integer isovalue. The Euler characteristic and triangle numbers are lower for the integer perturbed surfaces than for the non-integer surfaces, although the asymptotic table showed the least amount of difference when using the two different methods of isovalue selection. This is probably due to the fact that the asymptotic decider triangulates many of the cubes similarly, regardless of isovalue type. There also appears to be a best table

Figure 10: Surfaces from subject 1 using low lookup table. Isovalue of 41 in pink, 43 in blue, 45 in yellow. Region boxed in white is blown up to the right.



choice for the 4 tables tested with perturbed data values. The tetra table is the worst table when comparing χ values, although the other tables have mixed results. When using the perturbing method, the low table showed the best χ results for a majority of the subjects. The low table was also consistent in producing the lowest number of triangles, which can be seen in Figure 6. The mean maximum degree for the low, asymptotic, high, and tetra tables were taken over all subjects and isovalue selection methods. This property shows a reflection of table quality and is not dependent upon the isovalue selection method. The mean maximum degree for the low, asymptotic, high, and tetra tables are 12.16, 12.10, 15.77, and 14.12 respectively indicating that the low and asymptotic table performed best in this area. Thus, by looking at the χ values, triangle number, and maximum degree, we found that an algorithm using an integer isovalue, perturbing the data values equal to the isovalue, and using a lookup table which separates low data values gives the best overall performance.

One of the most surprising results was the outcome of surface area testing. In all cases the surface area ranking by table from lowest to highest was low, asymptotic, tetra, and high, as seen in Figure 7. Two-tailed paired t-tests were performed across each pair of tables using subject surface area values obtained from all isovalues and isovalue selection methods. Each table pair obtained a p-value less than 1E-10, showing that surface area differences are highly statistically significant. This indicates that surface area values will fall in a predictable pattern of low, asymptotic, tetra, and high in increasing surface area when using these four tables. The average percent increase in surface area across lookup tables from the least to greatest was 5.67 percent.

Surface area also appeared to increase with isovalue, as seen in Figure 8. Two-tailed paired t-tests were performed across each pair of isovalue levels tested (i.e. first tested value with third tested value) using surface area values obtained from all tables and isovalue selection methods. Our results indicated that surface area means are statistically significant, with p-values below 2.7E-50. This extremely low p-value can most likely be attributed to the almost constant rate of increase across all subjects and tables (see Figure 8). The average increase in surface area from lowest to highest isovalue is 18.15 percent.

We also found that, in general, χ values tend to increase with an increase in isovalue, although this is not always the case and the level of increase is not as constant as with surface area values. A subset of these values showing this trend is shown in Figure 9. This leads us to try to choose an isovalue that is as small as possible without sacrificing the deep penetration of the surface into the sulcal regions of the brain. Figure 10 shows an overlay of three surfaces generated from Subject 1 using the low table with perturbing for the first, third, and fifth isovalue tested.

DISCUSSION

The modifications to the original Marching Cubes algorithm have enabled us to reduce many of the defects often associated with this method of surface reconstruction. Using a consistent lookup table guarantees the absence of holes in the generated surfaces. The perturbation of data values equal to the isovalue eliminates the possibility of any non-manifold edges or vertices in our surfaces. Other methods could be used to determine the perturbation direction such as using only the 18 or the 6 closest neighboring data values, or an entirely different scheme altogether. Our method was chosen because of its simplicity to implement and its basis on neighboring values. Typically, this method also resulted in lower χ values than using the non-integer isovalues. The only remaining topological defect in our surface is handles, which can be removed by subsequent algorithms. In order to speed up downstream applications such as handle removal and flattening we wish to reduce the number of handles and triangles present in the surface by choosing an appropriate isovalue and lookup table. We have shown that our algorithm using our low lookup table, choosing an integer isovalue, and perturbing data values equal to the isovalue, performs better than with other consistent lookup tables or using non-integer isovalues. We have also shown that the surface area and number of triangles increases in a consistent manner with an increase in isovalue. The increase in these values may be attributed to the surface going deeper into the sulcal regions, although the consistency in the relative change in surface area is surprising. Our future work includes developing a fast topologically based algorithm for handle removal, as well as improving the quality of the generated surface mesh.

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