Homework 6 Introduction to Computational Finance Spring 2023

Solutions due Friday May 5, 2023

Answers to the homework problems and programming tasks should be submitted using the class canvas page.

You should submit pdf files. **Do not sent Word files or any other text processing tool's input file**.

As with all homework assignments you are allowed and encouraged to consult the relevant literature. You are also expected **to cite all literature that is used to generate your solutions and your solutions must make clear your understanding of the work cited.**

Programming Assignment

ODE methods and problems

Consider the following explicit numerical ODE methods:

- Forward Euler:

$$y_n = y_{n-1} + hf(t_{n-1}, y_{n-1})$$

- Explicit Midpoint:

$$\begin{gathered}
\hat{y}_1 = y_{n-1}, \quad f_1 = f(t_{n-1}, \hat{y}_1) \\
\hat{y}_2 = y_{n-1} + \frac{h}{2} f_1, \quad f_2 = f(t_{n-1} + \frac{h}{2}, \hat{y}_2) \\
y_n = y_{n-1} + hf_2
\end{gathered}$$

- Classical Explicit Runge Kutta 4-stage 4th order:

$$\begin{aligned}
&\hat{y}_1 = y_{n-1}, \quad f_1 = f(t_{n-1}, \hat{y}_1) \\
&\hat{y}_2 = y_{n-1} + \frac{h}{2} f_1, \quad f_2 = f(t_{n-1/2}, \hat{y}_2) \\
&\hat{y}_3 = y_{n-1} + \frac{h}{2} f_2, \quad f_3 = f(t_{n-1/2}, \hat{y}_3) \\
&\hat{y}_4 = y_{n-1} + hf_3, \quad f_4 = f(t_n, \hat{y}_4) \\
&y_n = y_{n-1} + h\left(\frac{1}{6} f_1 + \frac{1}{3} f_2 + \frac{1}{3} f_3 + \frac{1}{6} f_4\right)
\end{aligned}$$

(6.1.a) Apply the methods to the following initial value problems

$$f(t, y) = \lambda y, \quad y(0) = c \to y(t) = ce^{\lambda t}$$

$$f = \begin{pmatrix} f_1(y_1, y_2) \\ f_2(y_1, y_2) \end{pmatrix} = \begin{pmatrix} \omega y_2 \\ -\omega y_1 \end{pmatrix} = \begin{pmatrix} 0 & \omega \\ -\omega & 0 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \ \begin{pmatrix} y_1(0) \\ y_2(0) \end{pmatrix} = \begin{pmatrix} 0 \\ \gamma \end{pmatrix} \to \begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix} = \begin{pmatrix} \gamma \sin(\omega t) \\ \gamma \cos(\omega t) \end{pmatrix}$$

$$f = \lambda(y - F(t)) + F'(t) \ \ y(0) = y_0$$
$$y(t) = (y_0 - F(0))e^{\lambda t} + F(t)$$

(6.1.b) You should select various values of λ and ω, initial conditions, final time value, T, and functions for $F(t)$ and then run the methods with various fixed stepsizes.

(6.1.c) Integrate the problems with the various choices using different constant stepsizes and examine the behavior in terms of error (since you know the true solution the global error can be computed) and stability. Compare the methods in terms of accuracy for different stepsizes and observed rate of convergence as $h \to 0$. You should use total number of evaluations of f as the measure of complexity.

(6.1.d) Can you detect any evidence in the behavior that reflects the fact that the methods are of different order?

(6.1.e) Can you detect any evidence supporting the claim that a nonconstant sequence of stepsizes would produce similar error but with less complexity?

(6.1.f) Implement the implicit method Backward Euler using Forward Euler as a method to generate the initial guess $y_{n+1}^{(0)}$ on each step and functional iteration to solve the implicit equation for y_{n+1}. Does the implicit method improve the efficiency while preserving accuracy compared to the explicit methods?

Written Exercises

There are no written problems in this assignment.