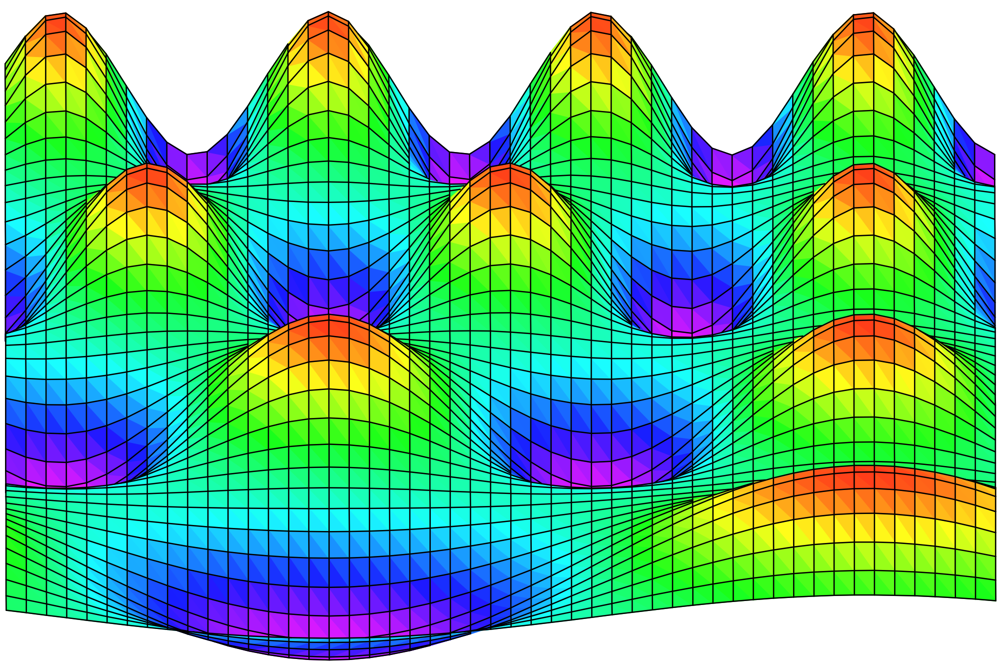
Computing Riemann Theta Functions

Consider a geodesic (a path that is locally length-minimizing) on a sphere. The coordinates of a particle that follows this geodesic are periodic functions of time, in this case the usual sine and cosine functions. Now replace the sphere by a compact surface with g holes (a Riemann surface of genus g). If for example $g = 1$, then this Riemann surface has the shape of a doughnut. Again follow a geodesic. Now the coordinate functions need not be periodic but still display patterns that are almost periodic. To a Riemann surface one can associate a Riemann matrix (or period matrix) and Riemann theta functions.

Theta functions were originally devised to solve the Jacobi inversion problem. It is known that all Abelian functions (generalizations to multiple arguments of elliptic functions) can be written as ratios of homogeneous polynomials of theta functions. This allows the expression of solutions of integrable systems in terms of theta functions. Such solutions are relevant in the description of surface water waves, nonlinear optics, etc. Because of these applications, we have developed and implemented algorithms for computing the Riemann matrix (see [1]) and the corresponding theta functions (see [2]). The graph below illustrates the oscillatory behavior of a theta function with $g = 2$.



References

[1] B. Deconinck, M. van Hoeij, *Computing Riemann matrices of algebraic curves*. PhysicaD, 152, 28-46 (2001).
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[2] B. Deconinck, M. Heil, A. Bobenko, M. van Hoeij, M. Schmies. *Computing Riemann Theta Functions*, Math. Comp. **73**, 1417-1442, (2004).
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