Dimensions in Mathematics
2006, Vol.26, No2, 55-62.

Scaling and Representing Exponential Relationships[3]

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The recent focus on data analysis and statistics has made graphs an important part of the elementary, middle and high school curriculum (Friel, Curcio, and Bright, 2001). All students in middle and high school grades are expected to create, transform, and analyze a variety of graphs (NCTM, 2000). In their review of research on graph comprehension, Friel et al. noted three levels of graph comprehension: the elementary level at which the reader reads information explicit in the data presented on a given graph; the intermediate level at which the reader interpolates and finds relationships in the data presented on a graph; and the advanced level at which the reader analyzes, extrapolates, predicts, or generates relationships implicit in the data presented on a graph. Friel et al. (2001) suggested a need to develop more classroom material across the levels of graph comprehension. For example, middle school students are exposed to problems that are mostly at the elementary level of graph comprehension. In this article, we present two activities to engage middle school students in work with graphs at the intermediate and advanced levels. These activities might also be used with high school students or modified for use with upper elementary grades.

Research shows that students from middle grades to college level demonstrate errors or misconceptions related to scaling while working with graphs (Dunham & Osborne, 1991; Mevarech & Kramarski, 1997; Padilla, McKenzie, & Shaw, 1986). For example, Dunham and Osborne (1991) found that students encounter problems in interpreting asymmetric scales or in transforming and scaling figures given on coordinate axes. As Leinhardt, Zaslavsky, and Stein (1990) concluded, scaling plays an important role in interpreting or constructing graphs.

Creating graphs and working with problems that require interpreting the data help develop students' graph sense (Friel et al., 2001). Identifying the components of graphs and understanding how these components are related and affect the readability of a graph are abilities that show the presence of graph sense. Friel et al. recommended that students in grades 6-8 should have experience with complex continuous data sets that have a large

[3] These materials were developed in part under a Mathematics and Science Partnership grant funded through the Florida Department of Education, *Students Transitioning toward Algebra* (grant number 371-2354B-4CH01), directed by Maria L. Fernández. The views expressed in this article are those of the authors and do not necessarily reflect the views of the DOE. The authors wish to acknowledge Lingguo Bu, Erhan Haciomeroglu, and Robert Schoen for their contributions to the creation and implementation of the activities discussed.

spread and variation. Furthermore, NCTM (2000) suggested that students in middle grades should model and graph nonlinear relationships as well as linear relationships.

In the following activities (see worksheets at the end of this article), students work with exponential relationships that involve complex continuous data. The first activity helps students develop an awareness of the importance of scales and the second activity provides an opportunity to choose appropriate scales to graph increasing and decreasing exponential relationships and use the information to make predictions. Both activities were developed as part of a collaboration between the Mathematics Department and the Mathematics Education program under the Students Transitioning Toward Algebra (STTA) project at Florida State University. The activities were implemented with 44 teachers of grades 4-8 mathematics participating in the project, some of whom subsequently implemented the activities in their classrooms.

Activity 1: Modeling Bacteria Growth

This activity provides an opportunity to examine a variety of graphs from a realistic scenario, helping students develop an awareness of the importance of scales and explore possible misuses of scales. The activity consists of two questions. The first familiarizes students with exponential growth of the bacteria and prepares them for the second question involving the merits or faults of eight different graphs created from the same bacteria growth data. The idea for this activity arose from our classroom visits as part of the STTA project. We observed students frequently making scaling errors or choosing inappropriate types of graphs when graphing data. Analyzing graphs that are appropriate and inappropriate representations of the same data and discussing the features of these graphs can help students correct their misunderstandings.

Here we describe how we implemented the activity with the STTA teachers. A similar procedure can be followed with students. We introduced the task through discussion of the bacteria growth. The teachers were then divided into groups of three to complete the bacteria growth table. Group members were asked to share their tables with the class, discussing patterns observed. Some groups expressed the number of bacteria (in 10,000s) as growing in multiples of 2 such as 1, 2, 4, 8, 16, and so on, while others represented the same quantities by using exponential notation, such as 2^0, 2^1, 2^2, 2^3, 2^4, and so on.

After sharing their tables and discussing how fast the bacteria were growing, the groups were asked to analyze the graphs in question 2. Each group discussed whether the given graphs were appropriate or inappropriate representations of the bacteria growth, the merits and faults of each, and the information each graph provided about the bacteria growth. Groups were randomly selected to share their opinions about each graph.

- The participants identified Graph 1 as an inappropriate representation of the bacteria growth data because the scale for the *y*-axis does not contain equally spaced intervals, a common scaling error.
- Graph 2, the bar graph, was seen as inappropriate for use in this context because the bacteria growth is continuous. This graph assumes the amount

of bacteria is constant during every 30-minute interval, doubling at the end of each interval rather than growing continuously throughout the hours.

- The participants' discussion of Graph 3 focused on the continuity of the data; the class agreed that the dots should be connected to represent the continuous growth of the bacteria.
- Groups agreed that Graph 4 represented a time versus bacteria growth rather than the requested bacteria growth versus time.
- Both Graph 5 and Graph 6 were thought to be correct representations of the bacteria growth versus time; however, both provided limited information about the bacteria growth. Graph 5 was thought to be appropriate to demonstrate the rapid growth of the bacteria due to doubling every half hour; however, it lacks clear information about the amount of bacteria growth over the first seven hours. Graph 6 provides limited information about the bacteria growth for the first 2 hours, giving the impression that the growth is somewhat linear when it is not.
- The participants considered Graph 7 to be appropriate, as well as useful for providing information about the increasing rate of change produced by the doubling of the bacteria.
- Graph 8 was thought to be appropriate and valuable for providing information about the growth of the bacteria over the first two hours of growth.

These eight graphs were created based on our observations about students' and teachers' errors and misconceptions while creating, transforming, and interpreting graphs. Often times, students are given data and asked to create a certain type of graph from that data. Asking students to analyze different graphs created from the same data with some graphs appropriate and others inappropriate, and having students discuss the merits and faults of each graph as well as the information each provides, can help them develop their graph sense.

Activity 2: The Growth of Water-Hyacinth

The second activity provides students an opportunity to represent exponential growth in tabular, symbolic and graphical forms in a 'real world' context. It involves exploring a hypothetical scenario of the growth of the water-hyacinth flower in Starke Lake in Florida. Water-hyacinths are prohibited in Florida waters because of the speed at which they multiply. As motivation for the activity, we showed a picture of water-hyacinth to the teachers and asked them what they knew about this flower. Comments shared included "Water-hyacinth infestations can prevent sunlight and oxygen from getting into the water", "Decaying plant matter reduces oxygen in the water", and "Possessing this flower in Florida is illegal".

The STTA teachers worked in groups on The Growth of Water-Hyacinth handout. Group members were randomly selected to share their groups' findings. In contrast to Activity 1 above, the table in this activity encourages the generation of formulas related to the growth of the water-hyacinth. For columns 3 and 4, some groups used fractions such as 1/200, 2/200, 4/200, and so on, whereas other groups converted these fractions to percentages. During the whole class discussion, we talked about the advantages of using fractions or percentages, the value of making connections between two representations of the same numeral, and students' difficulties related to appropriate choice of scales for graphing data that includes fractions.

To create the three graphs in question 2, the teachers had to determine appropriate scales for graphing their data, titles for their graphs, and labels for the axes. Moreover, they had an opportunity to work with both increasing and decreasing exponential relationships.

After groups answered questions 3, 4 and 5, each group presented their findings to the class on chart paper. Questions 3, 4 and 5 involve interpolation and prediction as existing data points are used to approximate expected outcomes between those points or beyond. Most groups used their graphs to answer these questions. For question 3, because the lake size is 200 acres, teachers found 200 on the y-axis of their 'Acres of water-hyacinth versus time' graph, and approximated the corresponding x-value by using the exponential curve they created. Other solutions included trial and adjust using the formulas found in question 1. Acres of water-hyacinth can be represented by $2^{n/2}$, where n represents the week number; 200 acres falls between 128 acres at 14 weeks and 256 acres at 16 weeks. So groups tried n = 15 weeks producing approximately 181 acres, n = 15.5 resulting in about 215 acres, and then n = 15.3 weeks results in approximately 200 acres. High school students may solve the equation, $2^{n/2} = 200$ to answer question 3 by finding $n = 2\log_2 200 \cong 15.3$.

Conclusion

The activities shared in this article enabled us to have valuable discussions with the STTA teachers about possible scaling errors, graph comprehension levels, and representing and analyzing exponential relationships. Activity 2 was a nice follow up to Activity 1; the teachers discussed scaling and common graphing errors in the first activity which helped them respond to questions in the second activity. However, reversing the order is also possible, depending on the needs of students. Teachers who would like to implement these activities may contact us if they have any questions. Larger formats of the graphs are available.

References

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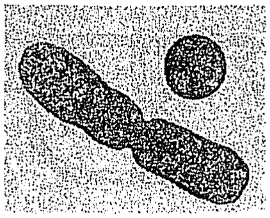
Activity 1: Modeling Bacteria Growth

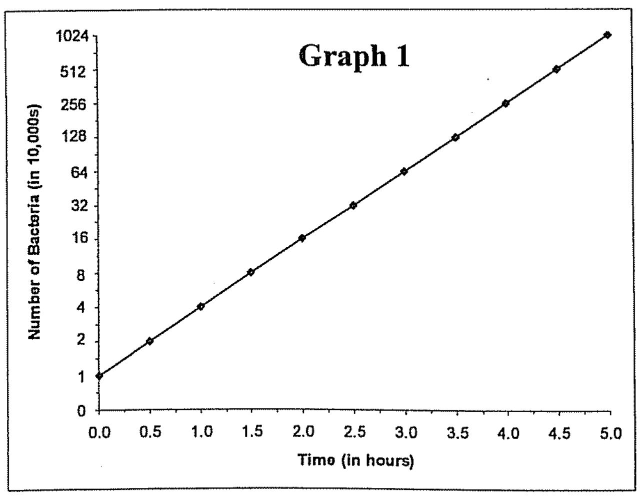


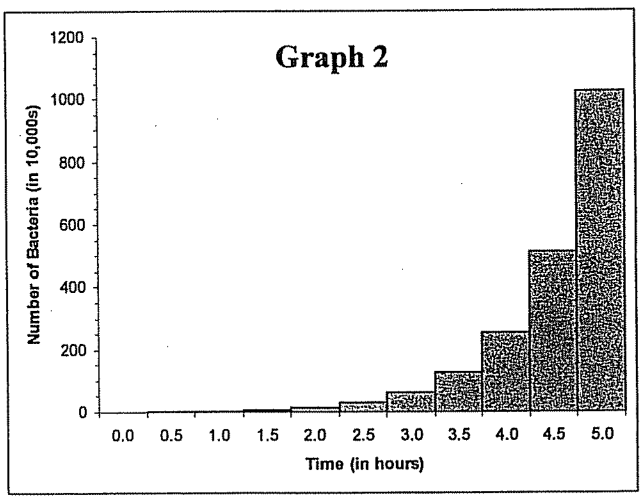
Bacteria reproduce very fast. Given the right conditions, with enough food and space, they will double in a few minutes. Suppose there are initially 10,000 bacteria in a certain colony and they are doubling their population every half-hour.

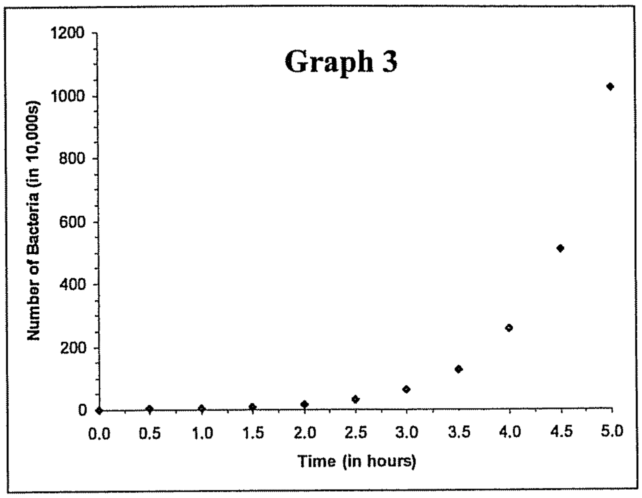
1. Make a table to show the growth of the colony for the next five (5) hours.

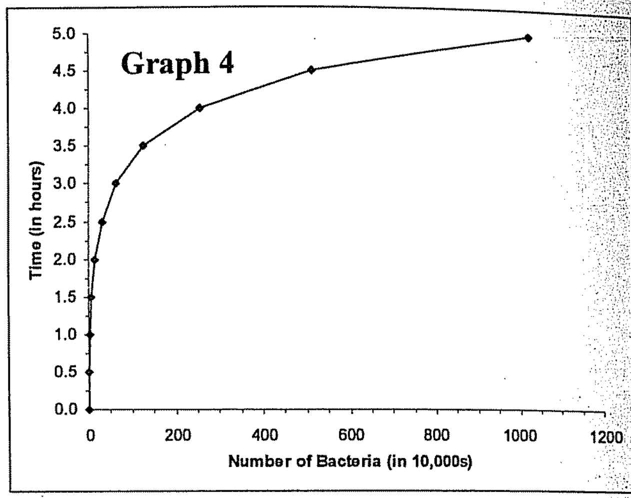
Time (in hours)	Number of Bacteria (in 10,000s)
0.0	1
0.5	2
1.0	
1.5	
2.0	
2.5	
3.0	
3.5	
4.0	
4.5	
5.0	

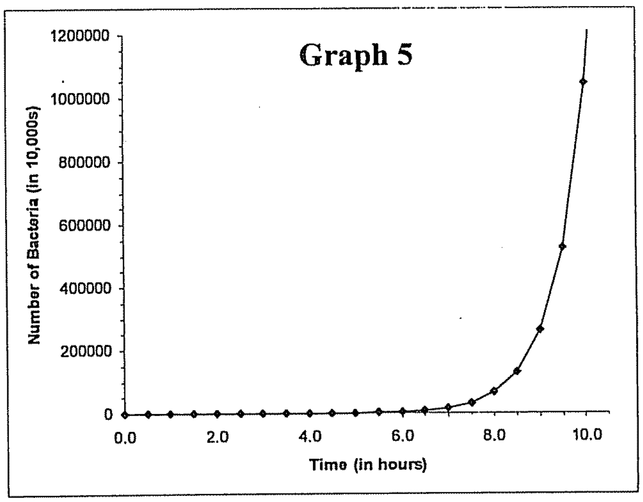
2. Determine whether or not the following eight graphs are appropriate as representations of the bacteria growth versus time. Discuss the merits or faults of each graph and the usefulness of the information each provides about the bacteria.

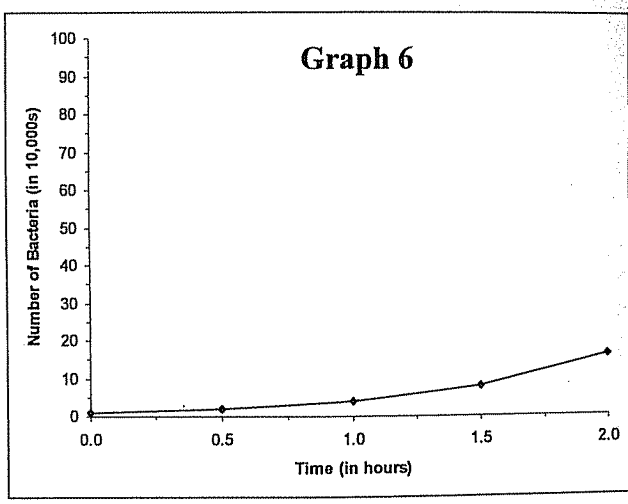


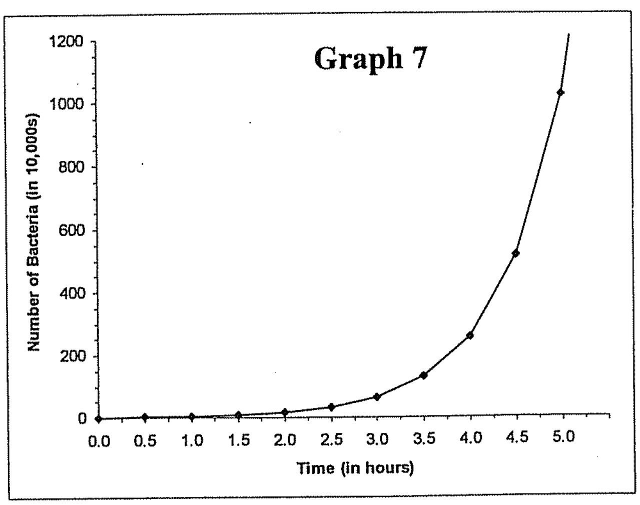


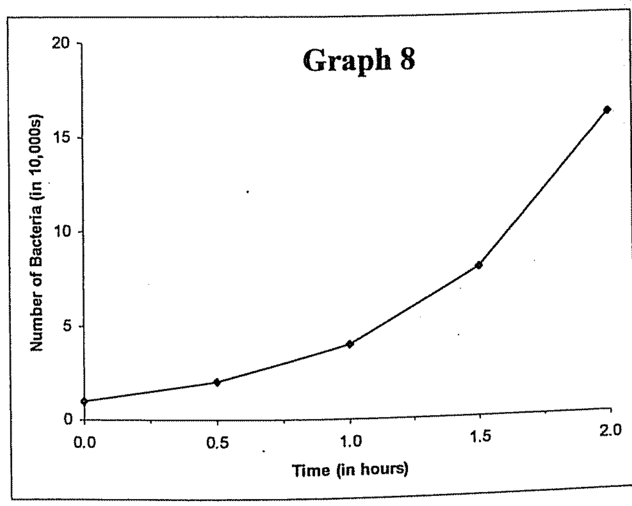

Graph 3
Number of Bacteria (in 10,000s)
1200
1000
800
600
400
200
0
0.0 0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0 4.5 5.0
Time (in hours)


Graph 4
Time (in hours)
5.0
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0 200 400 600 800 1000 1200
Number of Bacteria (in 10,000s)


Graph 5
Number of Bacteria (in 10,000s)
1200000
1000000
800000
600000
400000
200000
0
0.0 2.0 4.0 6.0 8.0 10.0
Time (in hours)


Graph 6
Number of Bacteria (in 10,000s)
100
90
80
70
60
50
40
30
20
10
0
0.0 0.5 1.0 1.5 2.0
Time (in hours)


Graph 7
Number of Bacteria (in 10,000s)
1200
1000
800
600
400
200
0
0.0 0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0 4.5 5.0
Time (in hours)


Graph 8
Number of Bacteria (in 10,000s)
20
15
10
5
0
0.0 0.5 1.0 1.5 2.0
Time (in hours)

Activity 2: The Growth of Water-Hyacinth

Starke Lake is a 200 acre lake in Orange County, Florida. Someone thought water-hyacinth was a pretty flower and he put some in Starke Lake. Today water-hyacinth covers one acre of Starke Lake. Research shows that the water-hyacinth population doubles in size every two weeks. You are a park ranger and it is your job to examine the effects of the water-hyacinth.

1. Complete the following table.

Week	Acres of water-hyacinth	Fraction of Starke Lake covered by water-hyacinth	Fraction of Starke Lake free from water-hyacinth
0	1		
2			
4			
6			
8			
10			
n			

2. Create a graph on graph paper that shows how the acres of water-hyacinth change over time. On another graph, plot the following graphs (using different colors): The fraction of Starke Lake covered by water-hyacinth versus weeks, and the fraction of Starke Lake free from water-hyacinth versus weeks.

3. How long does it take for the flower to completely cover Starke Lake? Explain your response.

4. When will half of Starke Lake be covered by the flower? Explain.

5. If Starke Lake was twice as big, how long would it take Starke Lake to be covered by water-hyacinths? Why?