Modeling Brain Anatomy with 3D Arrangements of Curves

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Introduction

We employ ***3D arrangements of curves*** to represent and analyze 3D shapes. The arrangements of curves may vary from fairly sparse – such as a collection of sulcal lines that coarsely approximates the shape of the brain – to very dense decompositions of the cortical surface into space curves. A ***shape space*** of such arrangements is constructed equipped with geodesic metrics derived from Sobolev spaces that can be used in conjunction with nonlinear curve registration techniques to quantify shape resemblance and dissimilarity. Although the metric measures global shape differences, deformation energies allow us to identify the regions where anatomical differences and similarities are most pronounced. The metric is applied to the analysis of configurations of sulcal curves associated with the left and right hemispheres of the brain. Examples are also given of geodesic interpolations between decompositions into space curves of surfaces representing the entire left hemisphere.

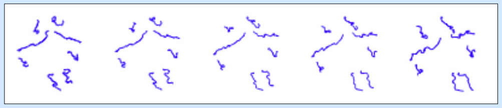

Figure 1. A geodesic between configurations of sulcal lines.

Representation and Metric

A configuration of n parametric curves $\alpha_j: [0,1]\to R^3$, $1\leq j\leq n$, is represented by a 3xn matrix A whose entries are the components $\alpha_{ij}:[0,1]\to R$ (i=1,2,3) of the curves. The Sobolev inner product $\langle f,g\rangle_1 = a\int f(s)g(s)ds + b\int f'(s)g'(s)ds$ induces an inner product on the space of 3xn matrices of functions given by

$$\langle A,B\rangle = \sum_{i,j} \langle \alpha_{ij}, \beta_{ij}\rangle_1 . \quad (1)$$

This metric accounts for shape geometry to first order, but higher order metrics can be defined similarly. As in classical Procrustes alignment, we first place the centroid of the configuration at the origin and scale the configuration so that $\|A\|=1$. This makes the representation insensitive to translations and scale and restricts A to the infinite-dimensional unit sphere (about zero) in the subspace of centered matrices. To calculate the optimal orthogonal alignment between normalized matrices A and B, let M be the 3x3 matrix whose (i,j) entry is the scalar $\sum_l \langle \alpha_{il}, \beta_{jl}\rangle$. If $M=USV^T$ is a singular value decomposition of M, then the optimal orthogonal alignment of B relative to A is given by $B^*=UV^TB$. Moreover, the geodesic shape distance is given by $d(A,B) = \arccos \omega$, where $\omega=\mathrm{trace}(S)$, and the geodesic deformation is realized by the path

$$\Lambda(t) = \cos(\omega t)A + \sin(\omega t)v(A,B), \quad (2)$$

with $v(A,B) = (B' - \omega A) / \|B^* - \omega A\|$.

The geodesic ***deformation energy*** is given by $E = \int \|\Lambda'(t)\|^2 dt = d^2(A,B)$.

Decomposition of Surfaces

To represent a spherical surface with a collection of space curves, we first construct a minimal distortion parametrization $\alpha: S^2\to R^3$ with the techniques of [1,2], and then decompose the surface using a collection of parallels on the sphere, as shown in Figure 2.

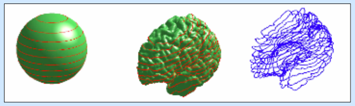

Figure 2. Decomposition of a surface into space curves.

Shape Alignment

In the case of sulcal lines, the correspondence between curves in different configurations is established using the optimal elastic alignment of their (unit) tangent fields as criterion. The registration technique is implemented via dynamic programming and represents a variant of methods previously used for single curves [2].

Parametric spherical surfaces were aligned with the techniques of [1,3] and a common set of parallels of S^2 was used to decompose them in a compatible manner, as indicated in Figure 2.

Examples of Geodesics

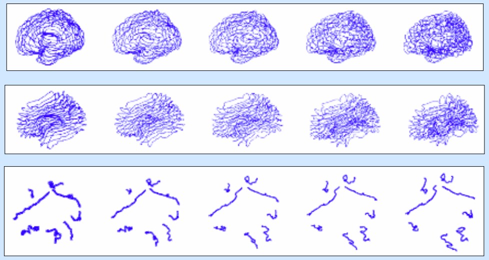

Fig. 2. Geodesic interpolations between aligned arrangements of curves.

Acknowledgements

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Local Shape Differences

The energy of the geodesic path (2) is given by $E(\Lambda) = \int_0^1 \|\Lambda'(t)\|^2\, dt = d^2(A,B)$.

Thus, the function

$$\rho_j(s) = \frac{1}{d^2(A,B)} \sum_i \int_0^1 (a\,|\Lambda'_{ij}(t)|^2 + b\,|\partial_s \Lambda'_{ij}(t)|^2)\,dt$$

can be interpreted as the energy density along the jth curve of the arrangement, which gives a measurement of the local contributions to the total shape distance.

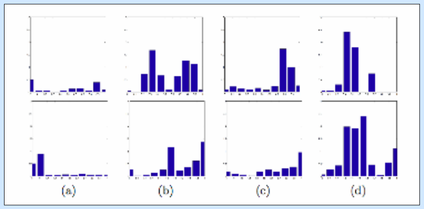

Fig. 3. Geodesic deformations between the left and right hemispheres of 2 subjects: (a) calcarine; (b) central; (c) superior frontal; (d) superior temporal.

Left vs. Right Hemispheres

a	b	k	Correct Classification
1	0	1	100%
1	0	3	100%
1	0	5	100%
1	1	1	79%
1	1	3	75%
1	1	5	92%

Experiments with 24 configurations of 4 sulcal curves representing the left and right hemispheres of 12 subjects, with weights a and b for the Sobolev metric. Decisions were based on the k-nearest neighbor classifier and a leave-one-out approach was used.

References

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