RATIOS AND PROPORTIONS: COMPLEXITY AND TEACHING AT GRADES 6 AND 7

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Introduction

This study relies on a previous survey that I conducted with 120 seventh-graders two years ago. A questionnaire presented for solution five ratio problems based on the same mathematical framework: working out a fourth proportional in combining the same type of data. But each problem referred to a different physico-empirical context (e.g. mixture and enlargement). For a given student, significant variations were found, from one problem to another, both in the success rates and in the procedures used. It seems therefore necessary to better take into account the physical context, and to classify the different types of ratio problems one student can face at this scholastic level, according to physical references. It seems also desirable to better organize, in the teaching, the articulations between and within the physical and mathematical domains. Now, in order to better articulate, it is first advisable to better separate, that is, better point out: differences between physical investigations (multi-sensorial with or without the real or virtual use of instruments) and mathematical ones (formal expressions and processing); differences between the diverse ratio problems on one hand and between the means of expression of rational numbers on the other hand. So that the complexity of ratio problems at Grades 6 and 7 is declined in three levels of separation/articulation. Kieren and Noelting (1980), or Vergnaud (1983) emphasized the importance of physical references in solving ratio problems. I just try here to specify these references and how the mathematical ways of expressing ratios match to them.

Hypotheses

At the considered scholastic level (grades 6 and 7),

1. The complexity of ratio situations can be described by two variables, one referring to the physico-empirical domain[1], the other to the mathematical[2], and the relations between and within their values.
2. Because of its physical and mathematical features, the graduated line, in a specific computer environment, allows to actualize the three levels of separations /articulations within and between the physical and the mathematical domains.

Methodology and Findings

Two groups of pupils were followed during two school years at grades 6 and then 7. The teaching sessions, in the group referenced as the Ful-experimental group, took into account the

[1] Values (the thinkable types of ratio situations): ratio of two heterogeneous quantities" (e.g. speed), "measurement", "mixture", "frequency", "enlargement", and "change of unit".

[2] Values (the considered semiotic registers in the sense of Duval – 2000): Graduated line, fractional and decimal writing.

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three levels of separation/articulation described in the introduction. Furthermore, the mathematical part of the teaching mainly relies, in a computer environment, on the graduated line, according to. In the second group, referenced as the Partial-experimental group, the teaching was based on the same corpus of ratio problems, but was led in a paper-pencil environment and without stressing the different articulations and separations. The findings show that variations (success rate and procedures used) are important from one group to the other. The Ful-experimental has a more complete evolution, which is a better acquisition of fractions and their use for solving usual proportionality problems.

WHAT COUNTS AS "PRODUCTIVE" DISPOSITIONS AMONG PRE-SERVICE TEACHERS

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Mathematics teachers' dispositions may influence the dispositions of their students, which presumably influence students' interactions with mathematics. It seems reasonable then to consider that teachers' dispositions may influence their students' interactions with mathematics. It is therefore important to try and understand the dispositions that pre-service teachers [PST's] may carry with them out into their classrooms. As teacher educators, we must clarify what composes a productive disposition for PST's before we develop more effective ways to nurture the development of their dispositions.

A productive disposition [PD] is defined to be a "habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy." (NRC, 2001, p. 116). In other words, according to the NRC (2001), a PD comprises components of a student's 1) beliefs and attitudes about mathematics, and 2) mathematics self-concept. A student's beliefs and attitudes about mathematics support the inclination to see mathematics as sensible, useful, and worthwhile. Mathematics self-concept constitutes the student's belief in his own efficacy and influences his belief that diligence leads to successful learning.

While the NRC (2001) defined dispositions in terms of students' attitudes and beliefs about mathematics, coupled with their mathematics self-concept, as mentioned above, it is less clear what these dispositions might look like in terms of how PST's talk about their beliefs, attitudes, and mathematics self-concept. In order to operationalize a productive disposition toward learning mathematics among PST's, I designed an interview protocol around these themes. The purpose of the interview is to examine the relationship between the connotations of students' responses regarding their attitudes and beliefs about mathematics, as well as their mathematics self-concept and their past achievement in mathematics. Eight first-year, elementary education majors at a mid-Atlantic state university were randomly selected for the interview. These students participated early in their degree program, while taking the first of three mathematics content courses for elementary education majors. Demographic data were compiled for each respondent. The interviews were audio taped and transcribed. The transcripts were analyzed thematically to build a composite disposition toward learning mathematics [DTLM] of PST's. Trends in the data were compared with the students' demographic data to distinguish a productive DTLM from non-productive dispositions. The hypothesis was that PST's whose responses communicated negative connotations about the sensibleness, usefulness, and worthwhileness of mathematics, as well as themselves as learners of mathematics would tend to have lower math SAT scores, grades in mathematics classes, as well as exposure to mathematics coursework beyond minimally required coursework than those students whose responses communicated generally positive connotations about the same topics. Data did not provide sufficient evidence to support the hypothesis unequivocally; however, trends in PST's responses suggest that PST's DTLM may be dependent on the content, in that, the connotations of some PST's responses regarding the sensibleness, usefulness, worthwhileness of mathematics, or themselves as learners of mathematics differed according to the mathematical topics being discussed.

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VIEWING PROFESSIONAL DEVELOPMENT THROUGH DIFFERENT LENSES: EXPERIENCES OF TWO MAJOR GRANT PROJECTS

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The National Council of Teachers of Mathematics (1991) recognizes the complexities of teaching mathematics well and the need to provide teachers with professional development (PD) opportunities that bridge theory with practice. Some characteristics of successful PD models are that they are long-term, school based, allow teachers to revisit their understanding of mathematical content, and encourage teachers to be active learners (Mewborn, 2003). This presentation offers how two major university grant projects are providing PD that employ these characteristics, among others, for teachers implementing reform-based practices in the teaching of mathematics.

Project Descriptions and Professional Development Opportunities

Project M^3: Mentoring Mathematical Minds is a $3,000,000, 5-year Jacob K. Javits grant funded by the U.S. Department of Education with the aim of designing challenging curriculum units, increasing attitudes towards and achievement of math for diverse grade 3-5 students with math potential, and providing ongoing PD. Ten schools of varying socioeconomic levels in CT and KY are participating. Teachers take part in an intensive two-week summer training session, PD inservices during the year, and weekly collaborations with a PD Team Member. *Beyond Access, to Math Achievement (BAMA)* is a $3,600,000, 3-year NY State Department of Education grant with the aim of raising teacher's math proficiency, increasing student math knowledge, and reducing achievement gaps among students. *BAMA* staff is serving 24 schools and 300 students in grades 3-8 in Syracuse City Schools, including 10 instructional support teachers providing PD.

Conceptual Framework

In an effort to better understand how both grant projects are addressing teachers' needs, Bolman and Deal's (1997) frameworks are being implemented. The purpose of their frameworks is to provide leaders with a set of lenses in an effort to consider and implement different approaches to address issues and better meet an organization's needs. The *structural frame* addresses the goals and formalized roles among an organization. The *human resource frame* views the organization as a social network where individuals have particular "needs, feelings, prejudices, skills, and limitations"(p. 14). The *political frame* sees the organization as an arena where groups and individuals compete for power and limited resources. Lastly, the *symbolic frame* considers how an organization also is a cultural phenomenon that has its own way of working and individuals or groups take on certain parts they are expected to play. This framework helps highlight the interactions between the universities and schools collaborating on *Project M^3* and *BAMA*.

Project M^3 and *BAMA* both have encountered accomplishments and challenges in their PD goals. Noteworthy accomplishments and challenges will be presented, including a discussion about the decisions made that addressed them, which frameworks were used to make sense of them, and our reflections about them. District-, school-, grade-, and classroom-level events will be discussed.

Conclusion

Both schools and universities are bestowed the responsibility of providing support for teachers to continue to grow professionally in their teaching of mathematics (NCTM, 1991). Suggestions on how to plan for and improve upon professional development will help facilitate this process for other professional development endeavors aimed at improving field-based mathematics education.

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ISSUES IN COLLECTING AND ANALYZING STUDENT REPRESENTATIONS OF DATA

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This poster session highlights the issues that arose in analyzing student-generated representations of data. The data was gathered from an elementary school while piloting a data unit from *Investigations in Number, Data, and Space*. In order to allow collaboration among the teachers, grades 1, 2, and 3 piloted a second grade unit, and grades 4, 5, and 6 piloted a fourth grade unit. In the beginning of the pilot, the research team determined a subset of activities involving written student work that would be collected to provide information on students' thinking of different aspects of data analysis – representation, description, comparison, and interpretation. This paper focuses only on representations.

The analysis of student work was an interactive process of individual analysis of student work, followed by group analysis, and subsequent revision of the analysis scheme. In the early stages of this process, only portions of the data were analyzed, until the group was sufficiently satisfied with the analysis scheme to warrant analysis of the entire data set. There were four main phases of progression of analysis scheme for student-generated representations. These phases will be presented at the poster session with detailed descriptions and sample student work. While the open-ended nature of the activities made these tasks a rich source of information about students' thinking, the wide variety of the responses made it challenging to create an analysis scheme that could capture the richness of the data. Different grade levels provided information on different aspects of representation of data, therefore the analysis scheme needed to describe and illustrate these differences in an efficient way. Two examples follow.

Consider the following set of data: 13, 1, 12, 0, 4, 13, 4, 4, 6, 0, 14, 12, 12, 6, 2, 4. A common representation for lower-elementary students involved drawings of cube towers to represent each value. In many cases, the towers were not ordered according to height (and thus data value), and in some cases, students chose not to represent the zero values. The research team considered the specific exclusion of zero values as significant, and thus created a criterion for it. However, after analyzing the representations of upper-elementary students (where this was no longer an issue), the team chose to simply include this under the more general criterion of *including all data*. Another issue arose when analyzing student-generated representations that generally resembled a bar graph. Most of the upper-elementary students using this general form of representing the data not only ordered the data, but also indicated holes in the data. For example, when representing the same set of data listed above, many of these students would leave a large gap between bars for the data values of 6 and 12, indicating that there was no data in that interval. In this case, the representation of this hole can significantly impact the way one makes sense of the data, thus the inclusion of this criterion in our analysis scheme was deemed

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essential. While representing holes had not been an issue with the initial lower-elementary student work involving cube towers, it did appear in later samples involving bar graphs.

The student work collected as part of this project was predominantly responses to relatively open-ended tasks, intended to prompt a variety of responses from students. Such a rich and varied data set is, by nature, difficult to analyze by reducing it to a small number of attributes. This process is made even more difficult by the attempt to create a framework that is applicable for student work in grades 1 through grade 6. The value in attempting to create such a framework is that it serves to highlight the issues that students face when they work to construct their own understandings of mathematical ideas (like representing data), rather than simply learning to apply taught understandings (like creating particular types of graphs).

INVESTIGATING VARIATIONS IN PROBLEM-SOLVING STRATEGIES FOR SOLVING LINEAR EQUATIONS

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To investigate variations in students' strategy development, this study engages students in problem-solving interviews, which have been widely used in research on mathematical problem solving (e.g., Star, 2001; Hunting, 1997). Specifically, study participants were prompted to share and explain their ideas before and after they solved problems. Data from these problem-solving interviews were used to identify, categorize and analyze students' developmental changes of strategies in problem solving.

Videotaped problem-solving interviews were conducted with twenty-three 6th grade students (12 males and 11 females). Students participated for a total of five hours over five consecutive days. Each student was given a pretest, twenty minutes of instruction, three one-hour videotaped problem solving sessions, and a posttest. In each of the three one-hour problem-solving sessions, students were asked a series of questions as they solved linear equations, including prompts to explain their choice of problem-solving strategies.

Of particular interest here is the level of sophistication of students' utterances relating to their written strategies. In order to analyze variations in students' strategies (as evidenced by students' utterances), several coding categories were employed. These categories include the consistency between utterances and written strategies, relations between actions and subgoals, goal-subgoal structure, certainty of utterances, speed of utterances, students' justification of strategy choice (e.g., quickest, most accurate, more familiar). Together these categories were aggregated to provide a measure of the sophistication of students' utterances.

There are three main results. First, students' utterances got more sophisticated as they gained problem solving experience. Students gradually increased the detail and rationale included in their descriptions of strategies as they engaged in more problem-solving practice. Second, students' written strategies got more sophisticated as their utterances got more sophisticated. Several students changed or added new written strategies when their utterances displayed more detail about their choices on problem-solving strategies. Third, students became more successful problem solvers (getting more correct answers) as their written strategies and utterances became more sophisticated.

The research reported here can extend our understandings of the developmental stages of problem-solving strategies for solving linear equations that have been highlighted in recent research on mathematical learning (Star, 2001; Catrambone, 1998).

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PRE-SERVICE MATHEMATICS TEACHERS' VIEWS OF THE NATURE OF TECHNOLOGY

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A philosophy of technology is important yet a neglected area in educational research (Flick & Lederman, 2003). The purpose of this study was to explore pre-service mathematics teachers' views of the nature of technology and how such views were related to their pedagogical beliefs in mathematics teaching and learning.

Theoretical Framework

Feenberg (2002) argues that there are three major schools of thought on the nature of technology. (1) An instrumental theory of technology argues that technology is merely a tool or device that is ready to serve the purpose of its user; technology is seen as neutral and apolitical. (2) A substantive theory of technology argues that technology represents an autonomous cultural system and fundamentally controls human thoughts and actions. Technology has been transforming our society to a more technically oriented system where values and questions are re-defined and solutions are directed to technical ones. (3) A critical theory of technology argues that technology is ambivalent in nature and the choice of civilization can be effected by human action. Humanity's future can be found in a democratic advance.

Research Context and Mode of Inquiry

The participants of the study were four pre-service secondary mathematics teachers. Data consisted of (1) a philosophy statement on technology use in education from each of the participants; (2) their electronic portfolio containing lesson plans, papers, project reports, etc. (3) three audio-taped semi-structured interviews in which participants elaborated their notions of the nature of technology and their perceptions of using technology in mathematics education.

Findings and Implications

The four pre-service teachers had homogeneous conceptions of the nature of technology and the role it plays in both education and society. They generally subscribed to the instrumental theory of technology described above. They viewed technology as a neutral tool that is under human's control. In education, they believed that technology could be used to aid learning or it could be used to expose kids to violence, depending on how teachers use it. They also believed that technology had the ability to empower teachers and enhance students' achievement. These findings are consistent with Fleming's (1992) study of 596 Canadian teachers' views on technology. These teachers overwhelmingly took an artifact or tool perspective on technology.

Constructed in this way, technology was treated as transparent. In their discourse on the role of technology in the teaching and learning of mathematics, the pre-service teachers in the study were not aware of the thought-mediating and culture-shaping characteristics of technology as observed by substantive and critical theorists of technology.

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THE USE OF ALTERNATE BASE SYSTEMS IN THE PREPARATION OF ELEMENTARY TEACHERS

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Understanding of the concept of ten is important for pre-service elementary teachers (PSETs). PSETs may not appreciate the complexity of this concept for students since they are familiar with it as adults. However, it is critical that PSETs see that student understanding of this concept (demonstrated by counting by tens, counting on by tens, and trading up and down in base ten) opens the door to many important mathematical concepts such as multi-digit addition, multi-digit subtraction, and multiplication. Pengelly (1990) states that "once all the ideas that characterize the number system are mastered, the structure [of the number system] becomes apparent, incorporating all the ideas that have gone before" (p. 376). Developing this appreciation and understanding of the aforementioned complexity is challenging for PSETs.

The purpose of this poster is to examine the progression PSETs make as they transition from content to methods courses. At Purdue University, PSETs are required to pass a series of three undergraduate (100 level) courses in mathematics during the first and second years of the program. While in the third year of the program, students are required to pass a methods course called Mathematics in the Elementary School. This course centers on using a problem solving approach to teaching mathematics. In both courses the use of base eight is implemented. In the first course of the mathematics series, different bases are introduced to help PSETs understand the difficulties their students may encounter while learning different operations in base ten. In the methods course, PSETs experience base eight while thinking about problem solving strategies that they may observe students using in the field experience component of this course.

This poster presents findings from a small scale research study of PSETs in these two courses. The questions guiding this study are: As PSETs progress through their mathematics content course into their elementary mathematics methods course, does their understanding of why the use of alternate base systems is included in the respective course curricula become more consistent with the stated course learning goals? To what extent does this dual approach, from methods and content courses, support PSETs in their development of this understanding? It is our intent that these questions will give us insight into the PSETs development of student understanding of the concept of ten and reveal connections PSETs are making between content and methods courses.

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EXAMINING PROSPECTIVE TEACHER SUBJECT MATTER KNOWLEDGE THROUGH STUDENT QUESTIONS

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Focus and Background

An essential part of teachers' knowledge that goes beyond specific topics within a curriculum is the subject matter that is to be taught. It includes, in very broad terms, the topics, facts, definitions, procedures or algorithms, concepts, organizing structures, representations, influences, reasons, truths and connections within the area of study and the connections outside the area of study to other areas. Leinhardt and Smith (1985) defined mathematical Subject Matter Knowledge (SMK) as the knowledge of "concepts, algorithmic operations, the connections among different algorithmic procedures, the subset of the number system being drawn upon, the understanding of classes of student errors, and curricular presentations" (p.247). This definition suggests that SMK has several influences that shape the learning and teaching of prospective teachers. Ma (1999) found several parallels between elementary teachers' SMK and the ability to function as an effective teacher within the classroom. A teachers' SMK influences both their actions in the classroom and their interactions with students. To further emphasize the importance of SMK, Shulman (1986) stated that a "teacher need not only understand *that* something is so; the teacher must further understand *why* it is so" (p. 9).

The Study

Thirty-one prospective teachers (PT) were asked a series of questions related to content that is usually taught in an Algebra 1 or Algebra 2 high school course. The questions were designed as student questions and the PT did not have prior knowledge as to the question being asked. At first the PT would be given 5 minutes to answer the question, as if the student had just asked the question in a class they were teaching. These responses were then collected and the PT were then allowed to form groups of 5-6 and then try and answer the question as a group. Then after about 10-15 minutes of discussion the groups were asked to present their ideas to the class.

Discussion

This poster will display several examples of answered questions. The purpose of the student questions were used as a reflective tool to examine the PT SMK and use these questions as opportunities to recognize that they need to know more than just the facts, terms, and concepts, but *how* and *why* they work.

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EFFECTIVENESS OF THE 'CHANGE IN VARIABLE' STRATEGY FOR SOLVING LINEAR EQUATIONS

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In this research, students' strategies for solving linear equations were examined. Of particular interest was the strategy referred to as "change of variable" or CV. CV was found when students rewrote terms such as 3(x+2) + 6(x+2) as 9(x+2). There are very few research studies which attempt to understand students' strategies to solve linear equations (e.g., VanLehn & Ball, 1987; Pirie & Lyndon, 1997). In prior research in this area, researchers have not commented on the use of the CV strategy to solve linear equations.

157 students who had completed 6th grade participated in five one-hour problem-solving sessions on linear equation solving. Students were given a pretest and then a short lecture (20 minutes) in which the researcher introduced four different steps for solving equations (adding to both sides of equation, multiplying both sides of the equations by the same constant, distributing, and combining variables or constants) to solve linear equations. After that, students worked to solve a series of linear equations for three one-hour sessions.

25% of students used CV at least one question throughout pretest and posttest. An analysis of the time that students spent solving each problem indicated that students who used the CV strategy spent less time than students who did not use CV. For example, the average time for all CV users was 3 min 41 seconds to solve CV questions throughout the pretest and posttest, while the average time for non-CV users to solve the same questions is 4 min 53 seconds, a difference that is significant. Not only did students who use CV solve problems faster, but they also used fewer steps to solve each problem, on average. While CV users typically solved problems in 3-5 transformations, non-CV users solved the same problems in 4-7 steps. The use of CV enabled solvers to solve problems both quicker and in fewer steps. Finding shorter and quicker solution paths is not only important for solution efficiency but it reduces the chance of error (VanLehn and Ball, 1987). Non-CV users had significantly higher rates of error on CV questions as compared to CV users.

CV is an example of an innovative strategy for solving linear equations, but it has received little attention in prior research on linear equation solving. This study represents an initial attempt to investigate the prevalence and use of CV among beginning algebra learners.

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MATHEMATICS COMMUNICATION AS EARLY FIELD EXPERIENCE

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Dewey (1904/1964) believed that in order to be able to hear and extend students' thinking in a given subject matter, ones own study of the latter should include reflecting on where students of different ages might be in relation to understanding the necessary pre-concepts and how one would build on this understanding. A century later, the U.S. Department of Education identified this kind of integrated learning – "developing teachers' mathematical knowledge in ways that are directly useful for teaching" – as the first of three areas for a proposed long-term research and development program (RAND Mathematics Study Panel, 2003), while new guidelines for teacher preparation also call for such integrated experiences (CCTC, 2003). However, given the sheer number of teachers prepared, limited personnel, and limited access to K-12 students, many mathematics teacher educators find it difficult to provide such structured early field experiences, let alone to integrate it into the mathematics content courses.

Online mentoring (OM), offered through Drexel University's Math Forum, provides a way to engage large numbers of future teachers in doing mathematics, communicating mathematics, and mentoring students who submit solutions to problems. Although OM does not let future teachers interact with students face to face, its ability to provide them time for reflection in the process of mentoring, is an advantage as an early field experience, since it allows novice teachers to focus on the mathematical content and on individual student thinking. As one future teacher stated, "I learned how to identify patterns of reasoning, differences in approach, and clarity of ideas." While student teaching, Jiang credited her experience with OM as the reason why she tried to anticipate different ways her students might solve a problem and why she encouraged them to communicate their thinking, both verbally and in writing. A year earlier, Jiang reflected on her OM experience as follows: "From the in-class discussion I saw that even my own classmates had different methods for finding the answer. This helped me understand that my students will also have different methods for finding the answers. Furthermore, the different methods were proven to me so now I can see for myself how and why [their] methods work."

Does participating in mathematics courses that integrate early field experiences emphasizing communication have a significant impact on the pedagogical content knowledge of teachers? The integration of online mentoring into mathematics courses for prospective teachers and a preliminary analysis of data related to the latter's pedagogical content knowledge, while in the course, and as novice teachers, will be discussed.

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SOFTWARE FOR THE YOUNGEST MATHEMATICIANS: CONNECTING QUALITATIVE, MULTIPLICATIVE AND ADDITIVE WORLDS WITH METAPHORS

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This poster focuses on children two to four years old, working with software modules. Each module is based on a metaphor mathematizing everyday actions. The recursive metaphoric process (Sfard, 1997) connects a source, consisting of more concrete, better understood images, and a target, which is the new, more formal concept being constructed. Children start from qualitative work, and then move to additive and multiplicative worlds within the same metaphor (Droujkova, In review). This approach allows systematic, "algebrafying" approach to different expressions of the fundamental mathematical ideas (Carraher, Brizuela, & Schliemann, 2000). Here are examples of modules, each coordinating several games and activities.

Hide-and-seek Equations

The image of *hiding* serves as a source of the metaphor that targets the idea of *unknown*. From the qualitative actions of figuring out *which* of different cartoon characters are hiding, children gradually move to quantitative actions of *how many* of similar characters are hiding. These actions, which can be additive or multiplicative depending on the game setup, are supported by the same hiding metaphor.

Grid Road Tables

The image of a grid of *roads* running at right angles to each other serves as a source for the metaphor that targets the ideas related to *tables*, such as covariation. Delivery cars children drag along each row or column "road" distribute qualitative features to be combined within cells, such as shape and color. The same metaphor is extended toward quantities combined in additive or multiplicative operations.

Function Machines

The image of a *machine* transforming the input serves as a source for the metaphor that targets *function*. Initial rules are qualitative, such as a machine transforming baby animals into adults. The metaphor of the machine coordinates additive and multiplicative operations. For example, "mirror machines" double or triple images symmetrically.

Fractal Power

The image of an *iterated splitting action* such as folding, branching or fragmenting (Droujkova, 2003) serves as a source corresponding to the target of unitizing and powers. The software supports the iteration of children's drawings, and the transition toward quantifying and representing the splitting actions.

There is an increasing need to support early mathematics education. Learning before the age of five may determine children's future success. Computer games can be a tool changing the ways young children and their parents or caregivers approach mathematics (Clements, 2002).

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A STUDY ON THE USE OF NETWORKED TABLET PCS IN THE ELEMENTARY SCHOOL CLASSROOM

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Research on networked Tablet PCs (Simon 2004) and other classroom networking devices (Kaput 2000) has focused on experiences in mathematics or computer science classes at the high school and college level. Although elementary school students have different needs from their older counterparts, they may also benefit from this emerging technology.

This preliminary study looks at two small K-8 Charter schools using Tablet PCs with wireless connection, enhanced with the software package Athena® (excelleworks, Inc.; Red Bank, NJ). Tablet PCs are laptops with the usual functionality, but with the additional capability of being used with a stylus pen as an individual whiteboard. The Tablet screens lay down flat so it is not unlike writing on a piece of paper. The Tablets are connected with a wireless network, and Athena allows the teacher, on his/her computer, to see each student's screen as they work. The teacher can use this copy of the student's screen to annotate his work and make comments and/or suggestions, which the student then sees on his own Tablet. Also, an individual student's work can be easily displayed as an overhead, via the wireless connection.

Teachers can use prepared lessons provided with the software, create their own lesson ahead of time, or use Athena® as they would use a set of whiteboards. As the class moves through each screen of a lesson, previous screens are saved for future reference. Thus, in effect, the Tablet PC is being used as an electronic notebook.

The research question is: To what extent are Tablet PCs being used in elementary classrooms to enrich the mathematical experiences? We look at ways in which the technology is used effectively towards this goal and outline ways it could be use more effectively. Informal discussions with teachers and administrators at the schools indicated more initial success at the younger grade levels. Thus, as a starting point the study focuses on Tablet PC-based Math lessons in Kindergarten through 3rd grade classes.

Observations take place once a week for several weeks; the tasks of the lessons vary. Preliminary results suggest:

- Students as young as Kindergarten use all the options offered with no difficulty.
- Students are attracted to the use of the technology and are focused on lessons.
- Teachers do not take full advantage of the capabilities of the technology, e.g., they often miss opportunities to display student's work to demonstrate different ways of thinking.

We will report on follow-up interviews with each of the classroom teachers which include questions focusing on why the teachers did certain things and why they did *not* do certain things during the observed classes. We will also report on teacher expectations, discoveries, and frustrations with the new technology.

An understanding of current practices may enable researchers to influence both the development of this emerging technology and the role it plays in elementary mathematics education.

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GENDER DIFFERENCES IN CHILDREN'S ARITHMETIC STRATEGY USE AND STRATEGY PREFERENCE

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In recent years, research revealed gender differences in first, second, and third grade children's strategy use for addition and subtraction problems (Fennema et al., 1998). More girls than boys used modeling, counting, and standard algorithms, whereas more boys than girls used invented strategies that take advantage of place-value properties in the base-ten number system. The current research study was designed to replicate and extend the findings from Fennema et al. (1998). The replication study re-examined existing data collected by Hiebert & Wearne (1992) from 72 children as they progress from first through third grade. Only children's strategies for multidigit addition and subtraction story problems that required regrouping were reanalyzed. The extension study used data collected from 15 second-grade children to explore their strategy preferences and their rational for the preferences.

The findings from reanalyzing the Hiebert and Wearne (1992) data suggest that gender differences in strategy use exist but the size of the differences are smaller than those reported by Fennema et al. (1998). No differences were found between first grade boys and girls addition and subtraction strategy use. In second grade, 1) more boys (100%) than girls (70%) used invented strategies to solve multidigit addition and subtraction story problems, 2) boys used invented strategies more frequently than girls did with a moderate effect size of 0.69, and 3) boys obtained more correct solutions than girls when using invented strategies with a large effect size of 1.13. No differences were found between second grade boys and girls use of counting strategies or standard algorithms. In third grade, 1) slightly more girls (96%) than boys (87%) used standard algorithms to solve multidigit addition and subtraction story problems, 2) girls used standard algorithms more frequently than boys did with an effect size of 0.55, and 3) girls obtained more correct solutions than boys when using standard algorithms with an effect size of 0.57. No differences were found between third grade boys and girls use of counting or invented strategies.

Findings from the exploratory extension study of 15 second graders showed most children preferred easy-to-use strategies. However, ease of use meant different things to different children. Several children claimed a counting strategy using manipulatives was easier because the demand on their memory was minimal and they could see and touch the items that need to be counted. Some children thought using the standard algorithm was easier because they knew the procedure, could keep track of sums or differences by writing down quantities, and it was faster than counting cubes. One child believed using an invented strategy was easier because it was less time consuming than counting cubes and it required less physical labor than writing an algorithm. In this small sample, the children's explanations did not appear to be related to gender but rather to their understanding of place value. Whether or not understanding of place value is related to gender will be investigated with a forthcoming study using a larger sample.

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TEACHER LEARNING THROUGH THE STUDENTS TRANSITIONING TOWARD ALGEBRA PROJECT

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The Students Transitioning Toward Algebra project is a partnership between the Florida State University Mathematics Department and Middle and Secondary Education Department in collaboration with the North East Florida Educational Consortium [NEFEC], and six member districts identified as high-need in accordance with project criteria. The goals of Students Transitioning Toward Algebra include the following: (1) the enhancement of middle grades (4-8) teachers' ability to prepare students for success in high school mathematics, particularly algebra-based courses; (2) the development of a learning community of middle grades teachers knowledgeable in mathematics content and instructional practices consistent with national, state, and district standards and curricula, as well as the Just Read Florida! initiative; (3) the enhancement of middle grades teachers' use of technology and manipulatives for promoting students' mathematical development in their transition toward algebra; and (4) increased mathematics achievement of middle grades students.

Project participants are 46 middle grades (4-8) mathematics teachers selected in teams of at least two per school in vertical groups across elementary and middle schools. Project teachers participated in two weeks of summer institute and six face-to-face and web-based meetings across the academic year. A matching control group of teachers was selected in order to investigate teacher learning and the resulting effect on students of teacher participation in the project. Data was collected through multiple data sources including surveys, content and pedagogical knowledge instrument, classroom observations, and interviews.

Students Transitioning Toward Algebra is significant in various ways. It will provide insight into how teachers develop their knowledge and use of instructional practices in ways consistent with national, state, and district standards and curricula through collaboration with mathematicians, mathematics educators and curriculum specialists; how to facilitate the development of a learning community of teachers that can help other teachers develop instructional practices that facilitate middle grades students' transition toward algebra; and how these instructional practices impact on students' understanding of concepts that support success in algebra based courses.

Initial results reveal project teachers’ growth in algebraic thinking and the implementations of lessons that foster their students’ algebraic thinking. Teachers are broadening their view of the definition of algebraic thinking and our building confidence in their own knowledge and teaching in this area. Teachers report that their students are enjoying project tasks and are developing their own algebraic thinking.

YOUNG CHILDREN'S MATHEMATICAL PATTERNING

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Mathematical patterning is fundamental to the development of mathematics. Steen (1990), in fact, argued that "Mathematics is the science and language of patterns" (p. 5). The years prior to formal schooling (pre-compulsory education and care services) are widely recognized as a period of critical development where the salient role of patterning features significantly.

In a multi-case study children's engagement in mathematical patterning experiences was investigated as was the teachers' involvement in, and influence on these experiences. The study was conducted in one preschool and one preparatory year setting. These sites were typical learning environments for Queensland children in the year prior to compulsory schooling. Multiple sources of data were collected. These data comprised semi-structured interviews with each teacher, copies of their daily programs and video-taped observation of the classes. Ten episodes of mathematical patterning were identified and categorized as teacher-planned, teacher-initiated, or child-initiated. Two episodes were initiated by children and the other eight were guided by the teachers. The nature of the teacher intervention in the child-initiated activities was of particular interest. Frameworks were developed to guide the examination of these episodes, with these frameworks being informed by the conceptual framework of Stein, Grover and Henningsen (1996).

The findings of this case study suggest that child-initiated episodes containing mathematical patterning are productive learning occurrences. During unstructured play times, children initiated activities that explored repeating patterns, pattern language, and the elements of linear patterns. These episodes were rich opportunities where children shared, refined, and developed their knowledge of patterns. Thus, child-initiated experiences can be powerful learning opportunities with the potential to develop children's knowledge of mathematical patterning in meaningful contexts.

The findings also suggest that teachers' understanding of patterning as well as their engagement in, and influence on child-initiated episodes impacts significantly on the outcomes of the event. Teachers play a myriad of salient roles to assist the development of mathematical patterning. The role of the teacher in questioning, providing resources, being involved, and offering encouragement has the potential to enrich mathematical patterning experiences and extend the children's existing knowledge. Likewise, teachers' limited knowledge of patterning concepts and processes, and the confines of their teaching competencies can hinder the outcomes of patterning events.

The poster will illustrate some of the above findings and will include a focus on how teachers' intervention can either extend or inhibit children's development of mathematical patterning.

Many early childhood professionals now agree that children should be "guided if not taught" to do some mathematics (Ginsburg et. al., 1999). When teachers understand what to teach, when to teach, and how to teach, they can provide rich opportunities for children to engage in patterning experiences, and capitalize on child-initiated learning activities.

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EXPERIENCE AS A POWERFUL TOOL FOR MEANINGFUL LEARNING OF PROBABILITY

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Ordinary problems using standard algorithms do not enable students to understand probabilistic situations meaningfully. Like in other branches of mathematics, students learning probability need to be involved in authentic situations that motivate their way of thinking. Littlewood (1953) declared that a good mathematics riddle (or joke in his words) is worth more than a dozen fair exercises. Probability is very much connected to every day life, but the synthesis between determinism and uncertainty makes it difficult to understand. The theoretical models used in explaining probabilistic thinking sometimes contradict intuition, which is based on every day experiences. Our experiences are deterministic, not continuous, and usually not guided (Rokni, 2001). Using guided experiences concerning probabilistic situations which derive from authentic problems may result in meaningful understanding of probabilistic principles.

The Research Question

What are the sources of mistakes in solving probabilistic problems and how does experience help lead to meaningful understanding of the correct answers.

Subjects

16 pre-service junior high school teachers.

Instrument

A questionnaire with three authentic situation probabilistic problems:

a. What is the probability of finding at least two people whose birthdays are on the same date, among 30 random participants?
b. There are three doors. Behind one door there is a prize. You are asked to guess where the prize is. After you guess, one of the other two doors is opened and you see that the space is empty. Now you are given the opportunity to change your guess to the remaining door. Will this increase your chance of winning? If so, what is the probability?
c. You have 2 discs: one is red on both sides and the other is red on one side and blue on the other. You choose one disc at random, put it on the table and you see red. What is the probability that the other side of this disc is also red?

Procedure

Step I.

a. 15 of the 16 participants wrote that the probability of finding at least 2 people whose birthday is on the same date is very small and may be 30/365, because there are 365 days in a year. Only one gave the correct answer based on previous learning of such a situation.

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b. All the participants wrote that changing the guess does not increase the probability. The only difference is that the probability changes from 1/3 to 1/2 in both cases.
c. 15 of the 16 participants wrote that the probability that the other side is red is 1/2, because the are only 2 discs, one with red on both sides. Only one participant (not the same one who answered item (a) correctly) gave the correct answer using an intuitive explanation based on 4 sides.

Step II

After sharing the results with the subjects, the researcher gave intuitive-logical explanations using demonstration and modeling for the 3 problems.

Step III

Some of the participants experienced cognitive dissonance after hearing the explanation because it did not fit their intuition and/or past experience.
The researcher then performed 3 experiments, one for each question above. The students participated directly and individually in all the experiments:

a. Collecting birthday data.
b. Simulating the 3 door game.
c. Playing a game with 2 discs.

Results

After step 3 (the direct experience), all 16 participants were convinced about the correct answer and changed their way of thinking about uncertainty.

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INTERVIEW EFFECTS ON THE DEVELOPMENT OF ALGEBRAIC STRATEGIES

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This research explored the effect that interviews and the presence of an adult 'helper' had on novice algebra students' work solving linear equations. 84 students participated, one hour each day, in a weeklong summer 'camp' before entering seventh grade. On the first day, they completed a pretest and were introduced to four operations to solve algebraic equations (adding/subtracting to both sides, dividing both sides, distributing, and combining like terms). Students spent three days working through linear equations before completing a posttest on the last day. Of the 84 students, 23 were randomly selected to work individually beside an interviewer. These students performed similarly on several pretest measures as non-interview students. For each interview student, an adult prompted the student to explain his/her work, reasoning, and strategies before or after solving selected problems. The adult did not provide assistance in completing problems but supplied encouragement and prodding ("What do you think you should do next?" "Nice work!"). The other students worked individually on the problems, were not interviewed, and were essentially provided no feedback as they worked.

The literature is clear on how the presence of an adult helper, even one who does not provide explicit help but merely words of encouragement, impacts student learning. Studies have documented the positive effect an adult helper can have in one-on-one learning situations (e.g., Bloom, 1984). In addition, the self-explanation literature (e.g., Chi, Bassok, Lewis, Reimann, & Glaser, 1989) suggests that students who are asked to verbalize or explain their problem-solving steps are more likely to develop deeper knowledge. Even when students are not self-explaining but merely describing the reasons behind their choice of strategies, beneficial effects have been found (Aleven & Koedinger, 2002; Stinessen, 1985).

However, in this study, students who worked with an adult did not benefit as much as other students. Interview students were *less* likely to get three of the eight post-test problems correct; a similar, although not significant, trend was observed with the remaining problems. Interview students used *more* problem-solving steps, and were therefore defined as less efficient, in correctly solving two post-test problems; a similar trend was seen for the other problems. These results raise questions about the benefits of self-explanation and an adult presence. Additional work should explore whether interviews may actually lessen algebra students' efficiency and their likelihood of solving problems correctly, particularly for novice learners.

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ENGAGING MATH FACULTY IN TEACHER PREPARATION

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The process of preparing prospective teachers needs to engage experts in the content areas, usually housed at the Colleges Arts and Sciences. Most of the time, the College of Education and the College of Arts and Sciences do not work collaboratively in this process but both groups could bring to the table important insights for the successful preparation of teachers. At The University of Alabama a group of faculty from both colleges has been working for three years, redesigning courses, adapting materials, and engaging other faculty in this endeavor. With the support of the Mathematical Association of America (MAA) through the Preparing Mathematicians to Educate Teachers (PMET) program, we have sponsored two workshops for faculty at math departments in our and other local institutions. The main purpose of these workshops is to have a dialogue among math faculty and math educators about courses for pre-service elementary school teachers. Additionally, we have two more goals, to get more faculty at our institution involved in teaching these courses and to provide us with feedback on the design of the courses. Both one-day workshops have been organized with the same structure. The topic and the relevant literature are presented in the morning, followed by a demonstration class with volunteer students from the teacher education program and a discussion of the lesson. In the afternoon the discussion is more open, having more input from participants and creating an opportunity for school teachers to share their experiences. The first workshop was held in Spring 2004 with 23 participants. The main topic for this workshop was the first course designed for pre-service teachers. The content of the course is Numbers and Operations. We used this workshop to present the recommendations from CBMS-MET report and some of the leading research in the area of teachers' content knowledge. We also discussed the role of the teacher in a classroom that promotes understanding and the demonstration class showed how this role could be modeled in a teacher preparation course. The second workshop was held in Fall 2004 with 30 participants. In this case the main topic was the use of technology in the preparation of prospective teachers. The two software programs used in the other two courses for teachers were presented with examples of how we use them in the courses. Geometer's Sketchpad and Fathom are used in the Geometry and the Data analysis courses. We discussed the technology standard from the *PSSM* (NCTM, 2000) and research related to the use of technology in math courses. Again, the demonstration class showed how the use of Geometer's Sketchpad supports student's development of mental images of geometric shapes. Teachers who participated in the Data analysis course shared their experiences and the influence the use of technology had in their understanding of statistical concepts. Both workshops were very well received by the participants and the evaluations show the need for more activities like these. As a result of the workshop, we have one new faculty member teaching the courses for prospective teachers and two ready for next fall. We have also organized a cadre of faculty from other local colleges and universities interested in teaching these courses. One of the projects for the near future is to provide all the resources and training for offering the courses at their institutions.

MULTIPLE SOLUTION STRATEGIES FOR LINEAR EQUATION SOLVING

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Although an algorithm (referred to here as the "standard algorithm" or SA) exists for solving linear equations, its use does not always lead to the most efficient solution (VanLehn & Ball, 1987; Star, 2001). For example, several possible solution strategies for solving an equation are shown in Table 1:

Table 2: Several possible solution strategies for a sample linear equation

Strategy I	Strategy II	Strategy III
3(x+2) + 5(x+2) = 8 3x + 6 + 5x + 10 = 8 8x + 16 = 8 8x = -8 x = -1	3(x+2) + 5(x+2) = 8 8(x+2) = 8 8x + 16 = 8 8x = -8 x = -1	3(x+2) + 5(x+2) = 8 8(x+2) = 8 (x+2) = 1 x = -1

The first strategy is the SA. The second strategy ("change in variable" or CV) uses an alternative in which (x + 2) was treated as a unit and then combined in the first step. The last strategy ("divide not last" or DNL) uses both CV and another transformation in which the equation is divided by 8 as an intermediate step, rather than as a final step (as is the case in the other two strategies). We will consider strategy III (which uses both CV and DNL) to be the most efficient, given that it involves the application of the fewest transformations. Of interest in the present research is how students learn to use and be flexible in their use of multiple strategies for solving linear equations. In this study, we were particularly interested in the effect of direct instruction of multiple strategies on students' ability to be flexible.

Method

153 sixth-grade students participated in the study. In the first one-hour session, students completed a pretest and were then introduced to the steps that could be used to solve equations. Students then spent three one-hour sessions working individually through a series of linear equations (similar to the one in Table 1). In the last session, students completed a posttest. Half of the students received an eight-minute presentation on how to use CV and DNL strategies (the "strategy instruction" or SI condition). The other half of students saw no examples of solved equations (the "strategy discovery" or SD condition).

Results

Although the SI and SD conditions had a similar effect on students' use of SA, *all* of the students who used CV in the post-test were in the SI condition. However, even after receiving a demonstration of the most efficient strategy (strategy III), *all* students who used CV only did so using strategy II. We interpret these results to suggest that students were able to initiate the most efficient strategies only through direct instruction, which is consistent with work by Schwartz and Bransford (1998).

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A VISION OF A THREE-DIMENSIONAL RE-CONCEPTUALIZATION OF MATHEMATICAL KNOWLEDGE

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Traditionally, mathematical knowledge has been classified along a single dimension: procedural knowledge vs. conceptual knowledge, as laid out by Hiebert and LeFevre in 1986. I propose that a three-dimensional model is more appropriate for classifying students' mathematical knowledge.

Hiebert and LeFevre defined conceptual knowledge to be knowledge that is rich in relationships; "A unit of conceptual knowledge cannot be an isolated piece of information; by definition it is a part of conceptual knowledge only if the holder recognizes its relationship to other pieces of information" (1986, p. 4). On the other hand, they defined procedural knowledge to include knowledge of the formal symbol representation system and of the rules, algorithms, or procedures for completing mathematical tasks. Hiebert and LeFevre suggested that the primary relationship between procedural knowledge units is "after," indicating that procedural knowledge is comprised of sequences of linearly related steps. Hence, conceptual knowledge was viewed as understood (well-connected), while procedural knowledge was not.

Star (2000) argued that the traditional usage of the terms procedural knowledge and conceptual knowledge obscures the myriad ways procedures and concepts can be known. He added a depth dimension to Hiebert and LeFevre's (1986) classification system to account for his observations. The traditional definitions do not easily accommodate all units of mathematical knowledge. For example, it is difficult to categorize the memorized facts of mathematics (such as the definition of slope or the commutative property of addition) as conceptual knowledge. After all, conceptual knowledge is supposed to be understood; yet facts can be memorized without being understood. Consider also the long-division algorithm as a classic example of procedural knowledge. An advanced student might know not only how to do the procedure, but also when to apply the procedure, how to predict the answer, and how to interpret the answer in a meaningful way. Adding a depth dimension allows us to more precisely classify students' procedural and conceptual knowledge. Well-memorized (but disconnected) procedures and concepts are labeled shallow; well-understood procedures and concepts are labeled deep.

There is still another dimension of mathematical knowledge that is of interest. During students' initial interactions with a concept or procedure, their knowledge can be considered tentative. In this early stage, execution of procedures may be error prone and require great cognitive effort, and conceptual facts may be recalled slowly or inaccurately. In time, this tentative knowledge becomes automatized and efficient, so that procedures can be executed fluently and facts and connections can be recalled on demand. This suggests the need for a third dimension to account for students' developing aptitude.

Therefore, I propose three dimensions of mathematics knowledge: type (procedural vs. conceptual), depth (shallow vs. deep), and aptitude (novice vs. practiced). My poster presentation will include a visual representation of the proposed three-dimensional model, as well as additional examples to illustrate the model and my vision of how this model can inform mathematics research and everyday classroom practice.

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PRE-SERVICE MATHEMATICS TEACHERS' CONTENT TRAINING: PERCEPTIONS AND THE "TRANSFORMATION" OF MATHEMATICS KNOWLEDGE FOR STUDENT TEACHING

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The mathematical knowledge required for effective teaching is a topic currently receiving much attention (e.g., Wilson, Floden, & Ferrini-Mundy, 2001; Ball & Bass, 2000; Ball, Lubienski, & Mewborn, 2001). Correspondingly, questions are being asked about "how many" and "what kind" of coursework can best serve mathematics teacher education students (TESs) as they begin their journey to become increasingly proficient instructors. In an effort to gain insight into this important area of inquiry, we conducted a research study to examine TESs' course taking patterns, their perceptions and understandings of their formal training, and how this training was a resource for them during their student teaching experiences and their future work as full-time teachers.

Subjects for this study were a cohort of mathematics education majors (n=16) at a large public university. Data were collected in the weeks prior to and during the student teaching experience. The data collection included surveys, transcript reviews, and in-depth interviews about their mathematics course taking, feelings of preparedness to teach various subjects and topics, and their understandings of how their formal mathematics training served as a resource for their teaching. Additional interviews and classroom observations were conducted with four participants to further explore, in situ, relationships between their formal training and their teaching.

Analyses revealed a wide range of perceptions regarding the role of formal mathematical training across the cohort and the differential valuing of various aspects of the disciplinary knowledge they held. Importantly, TESs identified differences between their knowledge *of* the discipline and their knowledge *about* the discipline, as well as an array of "gaps," which they felt affected their current efficacy as teachers. Despite nearly identical coursework in mathematics, TESs' feelings of preparedness to teach particular subjects varied across students and from subject to subject. Connections between these perceptions and other factors, such as the TESs' mathematical attainment, orientation towards students' thinking, and vision of themselves as a teacher, are examined.

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LESSON STUDY: A CASE OF THE *INVESTIGATIONS* MATHEMATICS CURRICULUM

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Generally, in the USA, concerted efforts are under way to move students' thinking from an instrumental and procedural understanding of mathematics to a relational and conceptual understanding (Senk & Thompson, 2003). This study explores how practicing fifth grade teachers' past instructional experiences impact their present teaching practices especially when implementing the prescribed *Investigations* curriculum. This study views the instruction of mathematics as the negotiation of practices of school mathematics with the teacher as initiator. Negotiation in this study involves reasoning, interpreting, and making sense of mathematical meanings. The study employs negotiation of meanings in its theoretical considerations and employs analytic induction in its data analysis (Bogdan & Biklen, 2003).

A qualitative case study research design was used to explore the teachers' practice of *Investigations'* mathematics in fifth grade classrooms. Data were collected through lesson plans, classroom observation for a semester and through audiotape and videotape of lesson study meetings. Three lessons for the lesson study meetings were planned, and only two were taught. The participants identified the lead teacher who went ahead and planned on an agreed-upon *Investigations* lesson. Even though different teachers led in planning a lesson, they all discussed it at length beforehand. The discussions focused on several issues e.g. what page they were on with the *Investigations* curriculum, classroom climate, students' work, or on the teacher. The lead teacher then conducted the instruction of the lesson as others observed and took notes. Finally the teachers met to debrief on instructional and content issues.

This study found that lesson study is a powerful intervention that influenced how the participants implemented the inquiry-based instruction in their classes. Holding a positive perception of *Investigations* is important but this alone does not inculcate a teacher's ability to use inquiry-based approaches. There should be an in-build framework to support reform efforts for practicing teachers. A structured, stable, and supportive environment is healthy for in-depth learning, work, and professional growth. This study found that professional development programs that are teacher-led and immersed in actual classroom lessons are effective (Fernandez & Yoshida, 2004), as seen in how these teachers changed their practice as they engaged and committed themselves more to the lesson study meetings – the teachers' instruction reflected more of constructivist's learning perspectives.

Working collaboratively, raising questions, or just hearing what others suggest about a lesson makes the participants' rise above self. To effectively implement the *Investigations* curriculum in schools that have adopted it, this study found that teachers must collaborate with each other and establish new classroom instructional approaches.

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THE "INSERTION" ERROR IN SOLVING LINEAR EQUATIONS

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This proposed research investigates a particular phenomenon that occurred during a study of students' flexibility in solving linear equations (Star, 2004). 160 6th graders participated in five hours (over five days) of algebra problem solving. In the first hour, the students were given a brief lesson on four different steps that could be used to solve algebraic equations (adding to both sides, multiplying on both sides, distributing, and combining like terms). Students then spent three hours solving a series of unfamiliar linear equations with minimal facilitation. 23 students (randomly selected from all participants) were interviewed while working individually with a tutor/interviewer. On the last day of the project, students completed a post-test.

Analyses of students' work made apparent an interesting type of error, named "insertion", in 12 (7.5%) students' of which three were interviewed. The insertion error was evident when 2x + 10 = 4x + 20 became **4x –** 2x + 10 = **4x –** 4x + 20. Similarly, 2(x + 5) = 4(x + 5) became 2 **– 2**(x + 5) = 4 **– 2**(x + 5). Interestingly, this error has not previously been reported nor classified in the literature on linear equation solving (e.g., Matz, 1980; Payne & Squibb, 1990).

Out of the many proposed classifications of students' rule-based errors in computational or algebraic problems (Matz, 1980; Payne & Squibb, 1990; Sleeman, 1984), Ben-Zeev's (1998) classification is the most relevant here. Its context of solving unfamiliar problems is very similar to the context of the present research. In this framework, the errors are classified into two major types: critic-related and inductive. Critic-related errors are due to the students' failure to signal a violation of a rule while inductive errors are due to student's over-generalization or over-specialization of conceptual interpretations or surface-structural features of worked examples.

The interview transcripts of three students suggest that they have over-generalized the procedure of subtracting the same term on both sides in order to eliminate a term of a linear equation. As a result, two erroneous procedures are created – one that violates the subtraction law by inserting "TERM –" to both sides and the other that violates the distributive law by inserting " – TERM" in between a coefficient *a* and its associated term ($x + n$). However, they stopped making these errors after they were made aware of the violation of such rules.

The data analysis thus proposes to include the following into Ben-Zeev's classification: 1) another critic-based failure whereby prior rules can be suppressed by the desired effect of a new procedure, and 2) errors which are generated by the confluence of over-generalized rules and critic-based failures even though this may be rare in the case of the "insertion" error.

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AN INVESTIGATION OF TEACHERS' MATHEMATICAL KNOWLEDGE FOR TEACHING: THE CASE OF ALGEBRAIC EQUATION SOLVING

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Solving algebraic equations is a central topic in traditional school algebra curricula. Although there have been extensive studies on students' understanding of equations and equation solving, few has been conducted with mathematics teachers. The knowledge of equations and equations solving that teachers employ for teaching becomes a particularly important issue for inquiry when the function-based approach has been reshaping school algebra curriculum, teaching, and learning in the past decade, and challenging the conventional, formal rule-based approach to equation solving.

Meanwhile, several groups of researchers (e.g., Ball, Bass, Hill and colleagues, Ferrini-Mundy and colleagues) have devoted to conceptualizing and measuring teachers' mathematical knowledge for teaching, both in general and in particular content areas (such as number concepts and operations, reasoning and proof, and secondary algebra). Most of their published work is about developing the measures. More detailed findings are in progress in terms of the component and characteristics of knowledge for teaching in a branch or special area of school mathematics.

This presentation is a summary of the initial stages of the presenter's ongoing dissertation research on secondary mathematics teachers' knowledge for teaching with specific focus on algebraic equation solving. A central piece at display is a conceptual framework for examining teachers' content knowledge for teaching mathematical concepts and procedures. The framework is constructed based on a summary of related theories, a review of research on student understanding of equations and equation solving, a mathematical analysis of equation solving process, analysis of algebra curricula, and the presenter's own empirical experiences in working with mathematics teachers. It incorporates five dynamic and interactive components:

1. *Knowledge of the core*. The typical or formal definition of a concept, standard or general algorithm for a procedure, and the mathematical rationale underlying them.
2. *Knowledge of alternatives*. Alternative definitions of a concept, alternative strategies for a procedure, the contrasts and connections between the standards and the alternatives.
3. *Knowledge of connections*. Definitions and properties of the same concept, algorithms of the same procedure, that are taught at different levels of mathematical study; other concepts and procedures that are connected to the one in focus (horizontally and vertically).
4. *Knowledge of presentations*. Effective ways of introducing a concepts or an algorithm, explaining related mathematical ideas to a specific group of students.
5. *Knowledge of learners*. The concepts that students bring into classrooms; typical student conceptions (misconceptions, mistakes, and difficulties) in learning a certain concept and procedure; effective ways of probing and assessing student understanding.

After being introduced, the framework is applied to analyzing the details of the mathematical procedure in focus: algebraic equation solving.

Two research instruments are being developed: A set of multiple choice and written-response items combined and embedded in teaching and learning scenarios, and a semi-structured

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interview protocol. The presentation discusses some design issues and also demonstrates sample items and questions from the instruments.

THE POSSIBLE CURRICULUM: ENCOMPASSING INTENDED AND ENACTED VERSIONS OF MATHEMATICS CURRICULUM

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This poster displays analytic tools for understanding the subtle differences between the mathematical goals of reform-based curriculum materials and the mathematics that teachers actually draw out and teach in their classrooms. The tools, once applied, provide ways to usefully describe the *possible curriculum*—the range of valid mathematical goals that can be addressed through the same material, and different routes teachers and students could move along to fulfill the goals.

Drawing on and adding to current research on curriculum enactment, we focus on the design and implementation of a 7th grade mathematics "replacement" unit which was developed in the context of a statewide experiment in Texas, and consists of SimCalc technologies together with written curricula and instructional materials specially designed to meet Texas standards. Specifically, we examine the ways three experienced teachers implemented a lesson focuses on a problem situation that involves motion, constant speed, unit rate, and proportionality, and with which students are engaged in collecting information, generating and analyzing data table, and formulating functional relationship.

Based on a through analysis of all mathematical concepts and processes involved in the lesson, as well as a preliminary review of observation notes and videotapes for classroom teaching, we developed an conceptual instrument that demonstrate two basis ways of working on the data table (down the rows and across the columns), and four ways of describing relationships observed from the scenario (physical formulas, arithmetic equalities, proportions, and function rules).

While the students in all three classes eventually arrived at a linear equation describing the motion of the runner in the scenario, each of the teachers focused their lesson on a different aspect of proportionality, some that the developer had not imagined as part of the intended lesson. We choose not to view this as an implementation failure. Indeed, these teachers got very high and significant student gains from pre- to post-test. Rather, we use their enactments and the conceptual instrument to create a "composite image" of the possible curriculum. Once articulated, this image can be utilized to help teachers and professional developers negotiate the mathematical terrain of conceptually rich mathematics instructional units. This approach can also be used to inform the development and use of materials within research projects designed to bring innovations to a wide variety of classrooms.

The presenters will be available to discuss the implications of possible curricula for the mathematics education community — researchers, curriculum developers and practitioners. The original curriculum materials and computer software used will also be on display.

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UNSTRUCTURED VS. STRUCTURED FREE-WRITING IN HIGH SCHOOL APPLIED MATHEMATICS

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Free-writing, which can promote self-assessment, was incorporated as part of a larger study on writing in a grade 10 applied mathematics class. Modifications to free-writing were conducted the next year in a grade 11 applied class. Schoenfeld (1985) states that students often do not self-monitor and self-evaluate; hence, teachers need to provide such opportunities for students. Rolheiser & Ross (2000) believe that through self-evaluation, students develop self-efficacy and motivation, which can lead to confidence, greater responsibility, and higher achievement.

In the initial study, students engaged in weekly 5-minute free-writes of what came to mind about mathematics (i.e., no specific questions were provided). Data analysis revealed that reflections were similar throughout the study; namely, the importance of maintaining/raising their mark and the past week's progress. Only 1 student (out of the class of 12 who participated in the study) provided evidence of devising action plans. As the study progressed, entries became shorter and student resistance (e.g., "Do we have to do free-writing?") was more prevalent. In the third anonymous questionnaire, only 3 of the 10 students understood the purpose of free-writing.

Reflecting on the initial study, free-writing was too open-ended. Lacking purpose, the assessment was invalid. That is consistent with the students in Broadfoot et al.'s (1988) study, who found self-assessment difficult since criteria was vague (in Gipps, 1999).

For the second study, the first author developed 13 prompts to allow students to reflect and develop action plans. The purposes of free-writing, along with a rubric, were shared at the beginning of the study. Students selected 1 prompt and spent 10 minutes engaged in reflection. Unlike the first study, descriptive feedback was provided after each entry, with acknowledgment of what is good and questions or suggestions to help students advance their thinking. Students wrote a total of 8 entries. In the 2 take-home assignments, they completed reflection sheets to identify up to 4 action plans stated in their entries, evidence of implementation, and 1 additional plan to be implemented in the future. Students did not resist, and at mid-term, 16 of the 21 students stated free-writing is beneficial (1 student provided no response).

In this poster session, the prompts and rubric, along with students' writing from both studies, will be shared. Also, questionnaire responses from the two studies will be compared.

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USING MATHEMATICS STORIES TO UNDERSTAND WHAT ELEMENTARY TEACHERS BRING TO REFORM

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Math stories like can provide important insights into what teachers value with regards to teaching strategies, how mathematical understanding develops, and, more generally, what teachers like and dislike about mathematics. Much has been written about how teachers' *past experiences* affect how they teach (Ball, 1997; Smith III, 1996). Teachers teach as they've been taught in what Lortie (1975) describes as an "apprenticeship of observation." At the same time, researchers have often described the role of *beliefs* in teachers' decisions about how and what to teach. Beliefs about the nature of mathematics, beliefs about how students obtain knowledge, and beliefs about how teachers convey knowledge are some of the kinds of beliefs that have been written about (e.g. Ernest, 1988; Thompson, 1992). In this poster, we will present the use of *mathematics stories* as both a conceptual and methodological tool for understanding the complexity and contexts of teachers' beliefs and experiences and for describing the frameworks, or lenses, through which teachers are seeing, interpreting, and implementing mathematics education reform.

The mathematics story interview asks teachers to consider all of their experiences learning and teaching mathematics. Teachers are asked to identify several key events within these experiences, including the high point, low point, and any turning points in the story as well as any challenges they may have encountered. They are also asked to describe a positive and negative future for themselves and mathematics.

In particular, this poster will address the following research questions:

- What types of stories do elementary teachers tell about their experiences learning and teaching mathematics?
- How are patterns in these story types related to patterns in teachers' grade level, years of experience with mathematics education reform, and teaching practices in the context of reform?
- How can mathematics stories be used as tools for research as well as for pre-service and in-service teacher education?

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NUMERATION CARDS: INNOVATIVE CURRICULUM MATERIALS FOR THE PRIMARY SCHOOL

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Introduction

It is very important that young students explore mathematics concepts using a variety of concrete materials. They help students to have initial opportunities to explore for themselves followed by careful guidance into an understanding of the abstract mathematics involved. Student-participants attending an urban elementary school in Edmonton were introduced to the numeration cards to help them do subtraction that involves renaming in problem solving. The purpose of the study was to make the learning of compound subtraction visible to participants by way of base complement additions strategy (BCA) using numeration cards.

The Numeration Cards

Trying to research into a problem may sometimes result in other discoveries (Gyening, 1993). According to history of mathematics education, "equal additions" was an approach for solving subtraction that involves renaming but it was very difficult to make its teaching and learning visible by using manipulatives. In an attempt to objectify equal additions "base complement additions" strategy has rather been discovered by researchers using numeration cards. Numeration cards are a set of innovative curriculum materials that was used for problem solving in addition and subtraction. The target task, notwithstanding, the numeration cards were found to have other uses in problem solving: numeration, addition and subtraction of all types, place values among others.

Methodology

This poster reports on a descriptive qualitative study on effective teaching and learning of compound subtraction using the numeration cards. Pre-intervention, intervention and post-intervention design was used as main source of data collection; and tests, students' scripts and semi-structured interviews were the evaluation instruments for the study. Two students in an urban elementary school, grades 4 and 5, volunteered to participate in the study. Each student was initially tested with the same test items and then engaged in an interview to find out entry-level strategies for compound subtraction before the intervention. The pre-intervention, intervention and the post-intervention tests were identical for each has the same number of items for two, three and four digits, vertical and horizontal digits, money and word problems. The same set of questions was given to both participant for the pre-intervention, intervention and the post-intervention tests. The post-post intervention tests were different, but parallel in form. The structure of the interviews was open-ended and the questions were developed in order to focus on participants' thoughts. Interviews between researcher and the students were audio-recorded; the researcher took detailed field notes during sessions with the students.

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Findings

Findings from the interviews implied that the cards helped the students to have better understanding of operations on numbers. From the students the numeration cards were useful manipulatives for effective learning of both decomposition (borrowing) and base complement additions strategies for subtraction that involves renaming. The students continued that activities using the numeration cards engaged them and motivated them to learn mathematics by seeing and doing.

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THE PROCESSES OF LEARNING IN A COMPUTER ALGEBRA SYSTEM (CAS) ENVIRONMENT FOR COLLEGE STUDENTS LEARNING CALCULUS

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Introduction

This study is a qualitative case study focusing on the question "What are the processes of learning in a Computer Algebra System (CAS) environment for college students learning calculus?" The study is designed to research the impact on student learning of particular software available for mathematics education and aims to provide insight into the nature of learning in a technology-rich environment.

Motivation for the Study and Theoretical Framework

There is research on student achievement in examinations after they have used CAS during their course of study. However, there is a gap in the research on CAS in the area of investigations into the processes of student learning and students' development of concepts while learning using CAS. Among my research questions are How do students use technology? Does using CAS effect students' learning strategies? Do students use the opportunities afforded by CAS to experiment with mathematical objects? Do students using CAS develop a different epistemological sense of mathematics compared to students not using CAS?

This research employs two theoretical frameworks through which to approach student learning while using CAS. The Rotman Model of Mathematical Reasoning (1993) is used as a macro-framework for the place of technology in the learning of mathematics. This framework is useful for addressing the question of the effect of technology on learning by positioning technology in the activity of mathematical reasoning. The Pirie-Kieren Model of Mathematical Understanding (1990) is used as a micro-framework and as a lens through which to interpret and analyse specific learning episodes as they take place in the classroom. The two frameworks together provide a vehicle for understanding learners' mathematical activity, reasoning and development in a CAS environment across a period of time.

Design of the Study, Methods, and Results

The primary data I used for the study are audio and video tapes of students in a college course learning calculus using CAS software. These data are supplemented by interviews I conducted with the students. My study provides a detailed description of the process of learning mathematics with the use of CAS and the emergence of conceptual development arising from collaboration among the students in the collective interaction with the software.

My principal findings are that (i) that students are aware of multiple strategies facilitated by CAS but often do not implement those strategies very well, and (ii) that the framing of technology in the learning environment has a considerable effect on how students approach their work and can be a hindering factor on their willingness to experiment in their learning. My study also shows that the adapted Rotman Model of Mathematical Reasoning is an accurate model for understanding the place of technology in the learning of mathematics; and that the Pirie-Kieren Model of Mathematical Understanding can be fruitfully applied to learning in a CAS

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environment. The significance of my work lies in the provision of a far richer picture of the CAS classroom than has been available before.

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ONLINE DISCUSSION IN A MATHEMATICS CONTENT COURSE FOR PRESERVICE ELEMENTARY TEACHERS

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Mathematics content courses for preservice elementary school teachers aim to promote deep understanding and clear communication of mathematical concepts, as well as familiarize future teachers with multiple approaches to math problems. These goals are achievable through reflection upon course material and the sharing of interpretations; journal writing and group collaboration have been suggested for encouraging these activities in college math courses (Beidleman, Jones & Wells, 1995; Dees, 1983). Online discussion boards (ODBs) enable both collaboration and reflection as an extension of classroom activities. This study fills a void in the literature by focusing upon a college mathematics course, the needs of elementary teachers, and the connections between course content and ODB use. Studies in other academic disciplines have shown that ODBs can encourage students to reflect upon and develop deeper understandings of course material, collaborate with peers, strengthen communication skills, and improve academic performance (Hofstad, 2003; Wickstrom, 2003). These studies have documented positive student experiences and have provided course-specific ODB implementation strategies.

This research studies the use of ODBs in a mathematics content course for preservice elementary school teachers, examining ways in which students use the ODBs, the impact of online collaboration and reflection upon students' mathematical learning and understanding, and students' attitudes and perceptions regarding ODB use. Students' perceptions of the online technology and their methods of use are determined via analysis of survey responses and patterns of interaction extracted from the ODBs. Cognitive and metacognitive activity, as well as the social construction of mathematical knowledge and understandings, are examined within ODB postings using modified versions of the content analysis models proposed by Hara, Bonk, & Angeli, (2000) and Gunawardena, Lowe & Anderson (1997). These models are also used to analyze student responses to exam questions, examining the transfer of deep understanding of discussion topics and the improved clarity of explanations beyond the online forum. Survey responses and measures of academic performance are compared between a course section using ODBs and another conventially taught section lead by the same instructor.

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SUPPORTING THE MIDDLE SCHOOL MATHEMATICS TEACHER IN PURSUIT OF NATIONAL BOARD CERTIFICATION

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The North Carolina Middle Math Project (NCM^2) is a state-wide professional development project for middle school teachers funded by the National Science Foundation. It is a collaboration between the North Carolina Mathematics and Science Education Network (NC-MSEN) and the North Carolina Department of Public Instruction (NCDPI). The major goals of the project are two-fold: to improve mathematics education in middle school, and to retain and support teachers in their professional development. Nine participating NC-MSEN centers assembled a team of university mathematicians, mathematics educators, school district administrators, and mathematics teachers to carry out the goals of the project. NCM^2 created three graduate-level courses for teachers, focusing on the content areas of statistics and probability, geometry and measurement, and number and algebra. Approximately one hundred thirty teachers took these courses. NCM^2 teachers are using this coursework and other initiatives of the project in the pursuit of National Board Certification in Early Adolescence Mathematics. The NCDPI and the North Carolina State Board of Education recognize this certification as an indicator of an accomplished teacher. Many teachers are also applying these courses to requirements for a Master's degree at several of the participating universities.

Our qualitative study was based on eighteen teacher interviews and classroom observations. In 2003, nine teachers, one from each NC-MSEN center, were interviewed. A classroom observation was also conducted. The following year, nine different teachers were interviewed and visits were made to their classrooms. All eighteen interviews were recorded and transcribed; field notes were taken of the classroom observations. In 2004, all teachers participating in the project were surveyed to measure the degree of the project's impact on their National Board Certification process. Analysis of this data indicated that the three graduate courses developed by NCM^2 had an impact on the teachers' National Board Certification process. They claimed that their preparation was influenced by two major components of the courses offered through the NCM^2 project: increasing teacher content knowledge and providing teachers with standards-based tasks. Overall, the NCM^2 participants specified that they were better prepared for completing the Portfolio Entries than the Assessment Exercises. More detailed results from the interviews, classroom observations, and surveys will be shared during the poster session.

Projects that increase teacher content knowledge can have a beneficial impact on a teacher's professional development as he or she undertakes the National Board process. To achieve National Board Certification status, a teacher must demonstrate he or she knows the mathematics being taught. Projects like NCM^2 can provide these learning opportunities. Successful teachers demonstrate a second type of knowledge that is equally important: the understanding of how to communicate the mathematics to their particular students (Wilson, Shulman, & Richert, 1987). NCM^2 courses and activities not only provided teachers with standards-based tasks, but showed teachers how to implement these tasks with middle school students.

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MATHEMATICAL CONNECTIONS IN OPEN-ENDED PROBLEM-SOLVING ENVIRONMENTS

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Cohen and Ball (1999, 2001) emphasized that instruction is a function of interactions among teachers, students, and content as mediated by instructional materials. Instructional materials shape what teachers and students do through the problems they use, their development of ideas, and the representations they contain. Teachers use their knowledge of the content and experience with students to interpret the materials and mediate students' opportunities to learn. Students use their prior knowledge to comprehend, interpret, and respond to materials and teachers. Moreover, students' prior knowledge and responses to the materials and content help determine what teachers can accomplish. In this study, we attempt to understand these interactions by looking at two classrooms in which the instructional materials are the same, but the teachers and students are different. Specifically, we explore what previous experiences and understandings students relied on in solving one problem and what experiences and understandings the teachers wanted the students to elicit as they attempted to solve the problem. In short, our research questions were (1) How do students solve open-ended problems, specifically, what connections to prior learning do they make; (2) Do the approaches the students choose to take match those recommended by their teachers; and (3) How do student approaches impact the goals of the activity?

Data for this study were collected in two sixth-grade classrooms across two days of instruction for each class. The primary data source was videotape, with two cameras being used in the classroom: one following the activities of the teacher and one focused on student and teacher writing. Additionally, videotaped interviews with students from each class asked students to explain their reasoning as they solved this problem and to comment on segments of videotape from the classroom. Videotaped teacher interviews relied on classroom and student interview data to engage the teacher in prompted reflection on her teaching strategies and goal setting and analysis of student thinking. Data were analyzed using a fine-grained analysis of talk, hand gestures, and drawings. The mathematics problem of interest came from the *Connected Mathematics Program* (Lappan, Fey, Fitzgerald, Friel, & Phillips, 2002) and asked students to determine the fractional portion of land owned by each of several landowners in a fictitious town. In each of the classrooms, students were allowed a variety of exploration approaches and each teacher worked with a somewhat different goal.

This poster presentation will highlight the different approaches students took to solving the problem and the mathematical experience they drew on to solve the problem. It will also draw conclusions about the mathematics goals addressed by the different approaches the students took within the framework of the goals the teachers wanted the students to meet.

Lloyd, G. M., Wilson, M., Wilkins, J. L. M., & Behm, S. L. (Eds.). (2005). *Proceedings of the 27th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education.*

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INVESTIGATIONS OF HOW AN IN-SERVICE TEACHER VIEWS HERSELF AS A LEARNER

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What does learning mean for teachers who are learners in a course devoted to the reconstruction of mathematics curriculum? How do they think about their own learning? How do they act as a learner? Are they reorganizing their mathematical thinking? Do they make connections between their experiences in this class and their constructed mathematical realities whatever they are (von Glasersfeld, 1985)? How do they become a viable member in the learning community?

These were the questions that framed my interactions with a high school in-service teacher, Tamara, a student in the curriculum course I served as an assistant. Throughout my interactions with Tamara, I have developed series of questions. As Stake (1995) wrote, "the best research questions evolve during the study" (p. 33). Those questions were how Tamara recorded her learning experiences, and how she reflected about them during the course, and what kinds of things she was paying attention to when she was a learner in a class context. As Mason (2003) said, “how attention is structured is crucial to what can be noticed, and what can be learned” (p. 13), so how Tamara was talking about those issues were important clues about my model of her as a learner.

The course was taught at a southern university in USA as a 7-week summer graduate level curriculum course. The course instructor had a purpose of the students reconstructing the basics of middle and high school curriculum: emphasizing combinations, permutations, counting, multiplicative reasoning, unknowns, binomials, fractional operations, Pythagorean theorem, and quadratic formulas. During the course, students used Geometer's Sketchpad (Jackiw, 1995) and JavaBars to investigate problems. I met with Tamara seven times during this course for one-hour long sessions. Those sessions were video-taped and partially transcribed. Two of these meetings were for interview purposes, but the remaining five were sessions in which I investigated Tamara’s learning while also offering help for her homework.

During the poster presentation, I will discuss how Tamara viewed herself as a learner in this class, what kinds of responsibilities she took, and how she reconstructed mathematical concepts using technology. I will discuss Tamara as a learner using two problems she solved related to fractions where she was supposed to use the concept of “co-measure” that was developed in the class.

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DEVELOPING TEACHER PRACTICE THROUGH VIDEO ANALYSIS

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Professional development is one of the primary vehicles through which teachers are provided opportunities to develop their knowledge for teaching mathematics. This knowledge includes understanding how each of their students learns and being able to design lessons that support and build on these understandings. The importance of meeting all students' needs is supported by the Equity Principle put forth by the National Council of Teachers of Mathematics (NCTM, 2000) which states, "Excellence in mathematics education requires equity – high expectations and strong support for all students" (p. 12).

This pilot study takes place in an ongoing professional development program in which teachers are studying their at-risk students' problem solving. The goal of this professional development program is to develop interventions to help support the mathematics problem solving abilities of at-risk students. In this program, at-risk students are students identified by their teacher as having difficulty problem solving. The participant is a middle school teacher in an urban school teaching one of the reform curricula. The question driving this pilot study is: In what ways can analyzing videos of at-risk students' problem solving provide opportunities for teacher learning? More specifically, how might this analysis help teachers to better understand how their at-risk students learn mathematics? Also, how might this analysis help teachers to better understand the effects of their teaching on at-risk students' learning?

The main source of data is three interviews with the teacher. One interview occurred at a professional development retreat, the next interview occurred after I observed one of her classes that was videotaped as part of the professional development, and a third interview was conducted after she was able to view and reflect on the video.

Preliminary results from analysis of the interviews suggest that that the teacher found the video analysis instrumental in guiding her teaching. In an interview she mentioned that if it was not for the professional development she would not be teaching the curriculum the "correct way" (interview 2), that she would be teaching it more traditionally. She also stated that observing the video enabled her to realize that her students knew more then she was giving them credit for, and in turn she adapted her classroom practice based on this information.

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A COMPARISON OF LEARNING SUBJECTIVE AND FREQUENTIST PROBABILITY

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Ever since the publication of *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989), probability and statistics have been prominent in the K-12 curriculum. A recent summary of research on human judgment and decision-making under uncertainty (Shaughnessy, 2003) addressed how humans rely on certain judgmental heuristics. Research shows that children have a subjective approach of playing out hunches, beliefs, and intuitions about what might occur during a probability experiment, yet the school curriculum does not consider subjectivity.

The purpose of this study was to explore the subjective theory (de Finetti, 1974) of probability with children. A mixed methodology study included students in grades 4, 5, 6 (n=87) who were engaged in a teaching experiment to compare learning traditional probability concepts (n=44) to learning traditional and subjective concepts (n=43). Pretest and posttest scores were analyzed using a MANOVA, while researcher observations from classroom lessons, teacher journals, and researcher interviews with students were coded for themes. All students improved significantly in probabilistic reasoning ($p<.01$). The combined fifth and sixth grade experimental groups who were exposed to subjective probability concepts improved more than the traditional group students ($p=.096$). Qualitative data showed that students have beliefs about probabilistic situations based on their past experiences and prior knowledge. This research adds to a growing body of literature about probability and statistics, and suggests that exposure to subjective probability concepts enhances students' reasoning skills.

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TESTING THE GRADUATED LINE AS A SEMIOTIC REGISTER FOR RATIONAL NUMBERS

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Introduction

Among the three registers of expression of rational numbers, described in Adjiage's poster (also presented to PME-NA 2005), the graduated line, seen as a semiotic register (Duval, 2000), is central in the teaching/learning process of fractional and decimal expressions. In the physical domain, the graduated line, equipped with a single or a double regular scale, permits to represent and to solve any of six considered types of ratio problems (Adjiage, 2000; 2005). This poster reports a research that tries to explain some of the difficulties that pupils face in using this semiotic tool in order to express and process rational numbers.

Main Hypothesis

- Using the graduated line (**GL**) for expressing and processing rational numbers requires the mobilization of a complex thinking, which includes not only mathematical knowledge, but also capacities of spatial structuring necessary to select and to organize the relevant information.
- Actualizing these competencies depends on contextual factors linked to the task, and on individual factors as former experiences in using a graduated ruler, inhibitions in former mathematical learning …

Methodology

Twenty-one 7th-graders of the same class-room are presented with ten mathematical exercises. Five of these exercises require to express rational numbers (e.g. "*Drop* $\frac{4}{5}$ *on a graduated line segmented in tenths*") using a GL. The results are related to those obtained with psychological tests (Rey' Figure and GEFT) and with other mathematical exercises: identifying different expressions of a given fraction, representing a fraction on a segmented surface, and thus mastering the operating mode of the denominator and numerator (Streefland, 1991).

The mathematical test is followed by individual "explicitation interviews" (Vermersch, 1993) which have two goals. The first one is related to the task: students have to make explicit the procedures they used and the obstacles they encountered when solving the exercises. The second one is related to the former students'experiences in mathematics, including their learning inhibition (Ancelin-Schutzenberger, 1993; Metz, 1999).

Results

An implicative analysis (Gras, 1992) made it possible to select in the mathematical questionnaire some items so that succeeding these items tend to imply succeeding to others.

Global results at the GL items are correlated with the GEFT results: pupils who succeed in operating the graduated line tend to be those who are able to get free from perceptive factors.

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An analysis of the interviews shows that a persistent fixing to the decimal system inhibits the capacities for using the GL properly. That allows to make a new assumption: some pupils remain hung on the decimal system because of the persistence of the "percept" related to the decimal graduated ruler as a well-known measuring tool. Many pupils fail to master the graduated line because of this "percept" which functions then as a "Gestalt". This inhibits the implementation of their capacities to properly operate the numerator and denominator of a fraction, and thus makes them incapable to represent this fraction on a graduated line, even if they succeed to represent it on a segmented surface.

APPLY WHAT YOU KNOW TO A PROBLEM YOU HAVE NEVER SEEN: HOW AP STUDENTS APPLY STATISTICAL REASONING TO SOLVE A TASK USING EMPIRICAL DATA

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Statistical reasoning has been defined as how one makes sense of statistical information and makes inferences using statistical concepts (Garfield & Gal, 1999). Students who have been classically educated in statistical inference techniques such as hypothesis testing and confidence intervals may perform well in statistics classes but can they use their statistical reasoning when faced with making an inference based on empirical data that they collect? This study examined the statistical reasoning displayed by students in two US high school Advanced Placement Statistics classes.

The task used in this study was adapted from a research study with sixth grade students (Stohl & Tarr, 2002) in which students used a computer simulation (Probability Explorer, Stohl, 2002) to generate data to determine if a fictitious company produces fair dice and to estimate the probability of each outcome if the given die is not fair. Our analysis focuses on how students approached the task, the size of the samples they chose to collect, and whether they applied statistical inference techniques to provide compelling evidence or used other non-standard evidence. We also examine the students' prediction of the probability of outcomes of the die they are working with and what statistical reasoning the students employ in their estimation. To examine the effects of the computer simulation on students' statistical reasoning, a second class of students also conducted the same experiment with physical dice, which had been weighted. Each pair of students investigated an assigned company, made a decision on whether that company produced fair dice or not, and estimated the probability of each outcome. Each pair then presented these results to their classmates in the form of an oral presentation and some type of display of their evidence. The class was then able to ask questions about the presentation and the students had to support their reasoning.

The data for this study is currently under analysis, though preliminary data shows that although some students confidently complete the task, others struggle to apply the methodology they have learned in a practical sense using empirical data. Our poster will contain a description of the task the students had to complete, analysis of students' statistical reasoning using Models of Statistical Reasoning (Garfield, 2002), and examples of students' work.

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USING EPIDEMIOLOGY TO MOTIVATE ADVANCED MATHEMATICS

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The quality of secondary mathematics instruction has on the decline due to increase demand to meet national education standards. In response to this growing problem, the presenters developed and implemented a mathematics workshop for secondary mathematics teachers and high school students to stimulate interest in advanced mathematics. The workshop was designed to introduce mathematical biology, an emerging discipline within the field of mathematics, and to demonstrate innovative ways of teaching mathematics with graphical modeling software.

Mathematical modeling was the primary topic of the workshop in which the participants explored basic concepts in abstract algebra and graph theory. The participants were engaged in discovering applications of mathematics through a hypothetical epidemiological problem based on actual data. This experience fostered the educators' appreciation of innovative ways of teaching mathematics using advanced mathematics and mathematical modeling software, with an emphasis on integration of these topics into the Standards of Learning mathematics curriculum. In addition, the activities generated interest of mathematical modeling in the students, while maintaining relevance of advanced mathematics.

During this workshop, we found that the students extended their problem solving and communication skills. All participants established a connection between mathematics and biology and discovered novel applications of technology to solve mathematical problems. Through the creation of an intergenerational environment in which educator worked alongside student and illustration to creative approaches to instruction, the high school teachers developed novel methods to integrate technology and advanced mathematics concepts into the standards of learning mathematics curriculum.

The presenters will introduce the framework of the workshop, including a description of the mathematics topics and the mathematical software. We will also present the epidemiological problem given to the participants. We will close with a discussion of the implications of this outreach experience.

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GEOMETRY CONNECTING RATIO UNDERSTANDING: REPRESENTATIONS, STRATEGIES, AND DISCOURSE

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Research indicates that people perceive of ratios in two ways—between ratios and within ratios (Lamon, 1994). Grounding these ideas in rectangles was central to one teacher's ability to orchestrate sixth-grade students' problem solving and subsequent metarepresentational competence in a two-year design experiment investigating geometric similarity as a means to promote algebraic understanding (Boester & Lehrer, in press; Lehrer, Strom & Confrey, 2003).

Combinations of student work and classroom discussion document how students and teacher measured similar rectangle sides to visually and verbally connect seven successively abstract representations of ratio: a) paper strips for depicting linear measure, (b) physical rectangle cutouts, (c) equations, (d) ratio tables, (e) co-ordinate graphs illustrating linear groupings of similar rectangles, (f) stair-steps drawn between points along graphed lines, and (g) slope. Fractions, ratios, and quotients were integrated alternative forms of ratio (Lamon, 2001).

In each section, the poster demonstrates that using rectangles with each representation enabled mutually sensible or interanimated conversations (Seymour & Lehrer, submitted). As a whole, it is a story of a teacher orchestrating understanding of limitations of the discrete between-ratio strategy, and virtues of the continuous within-ratio strategy for determining the slope of a line of an infinite group of similar rectangles.

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PROBLEMATIZING WRITING IN THE LEARNING OF MATHEMATICS

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Objectives

The purpose of this poster is to (a) explore the importance of teaching students how to write mathematics, (b) identify some of the challenges to researching and implementing writing instruction in mathematics classrooms, and (c) suggest directions for research.

Theoretical Framework

Communication has long been recognized as being important in the learning and teaching of mathematics. Policy statements such as the *Principles and Standards for School Mathematics* (NCTM, 2000) suggest that communication should not only include spoken discourse, but also writing. Aspects of writing in the mathematics classroom, such as representing and symbolizing, have been heavily studied. Much less research has been devoted to studying broader aspects of writing, such as the writing of explanations, justifications, conjectures, and descriptions. Given the important role that writing can play in the development of understanding, and given the importance of being able to write mathematics as a requirement for participation in legitimate mathematical activity, it is important to study these broader aspects of writing.

Methods and Results

We conducted a literature review to assess the challenges currently facing the research and implementation of writing instruction in mathematics classrooms. We encountered three assumptions among researchers about writing that inhibit the research and implementation of writing instruction in mathematics classrooms: First, writing is merely a tool to learning and understanding mathematics, and not a worthwhile end goal in and of itself. Second, if students are able to successfully vocalize their understanding, then they will have not trouble writing. Third, if students cannot write, it is because they do not understand, and not because they may not know how to write mathematics texts.

In addition to these three assumptions, we noted three additional challenges to the research and implementation of writing instruction: First, there is currently no common written discourse in reform-oriented mathematics classrooms. Consequently, it is unclear exactly what writing should be researched and taught. Second, due to the lack of a common written discourse, current attempts to research and implement writing instruction often rely upon the use of written genres from outside of mathematics, such as poems, songs, stories, historical reports, or philosophical arguments. Such efforts may be misguided, because these different genre are not particularly effective for communicating and developing mathematics. Third, it is unclear how to explicitly teach writing, because attempts to help students to write may be interpreted by students as providing them with the ideal explanations or solutions they should be memorizing.

Conclusions

Research is needed to identify the types of writing that are being done in current reform-oriented mathematics classrooms, to judge if these types of writing are appropriate for learning

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and participating in mathematics, and to document how teachers are promoting these types of writing. Further research is also needed to identify other types of writing that are currently not being used in classrooms, but that could enhance learning and students' ability to participate in mathematical activity. We anticipate that the teaching and learning of mathematics will benefit as the field moves toward establishing a written discourse for students' mathematics and methods for facilitating fluency in this discourse.

ASSESSING PRESERVICE TEACHER HABITS OF MIND WHEN ATTEMPTING AND PLANNING A MODEL ELICITING ACTIVITY

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Focus of Study

The purpose of this preliminary study is to examine the actions of several preservice teachers when working on a model-eliciting activity, involving slope and compare their actions to their ideas about students' actions when planning a similar lesson.

Conceptual Framework

This research is based on the frameworks of Pirie & Kieren (1994) and Berenson et. al (2001) who propose that the growth of mathematical understanding and pedagogical content knowledge can be observed progressing through iterative levels. The theory, developed to offer a language for observing this progression, contains eight potential levels for understanding, the first five of which we use in this study: *primitive knowing, image making, image having, property noticing* and *formalizing*. Berenson expands the Pirie-Kieren model to account for tasks involved in the preparation of teachers. Our interest in this study is to compare the preservice teachers' knowledge of slope when solving a model-eliciting activity as measured by their attempts to fold back, and their awareness when planning lessons of this need in students. Lesh and Doerr (2003) refer to model-eliciting activities as those that involve making mathematical descriptions of everyday situations.

Methodology and Results

The researchers acted as co-teachers of a larger teaching experiment occurring over a 14-week mathematics education methods course, at a large southeastern university. Weekly class meeting were designed to encourage preservice teachers to use multiple representations within and between selected mathematical topics, all pertaining to proportional reasoning.

We found that these preservice teachers relied heavily on images and representations, marked by various levels of rigor, in order to complete their work. This lends itself to the Pirie-Kieren model (1994) in that *image making* preceded a more formal, symbolic result. These preservice teachers folded back to images that are more elementary to assist in their work. On the other hand, when preparing an introductory lesson on slope, a significant number planned to use their students' presumed knowledge of linear functions and coordinate geometry to teach a symbolic representation of slope, thus overlooking the need of their students to make images and notice properties, before formalizing.

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UNDERSTANDING TEACHING AND LEARNING OF FRACTIONS IN A SIXTH-GRADE CLASSROOM

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Teaching and learning of mathematics have been studied separately most of the time in the literature. There is research either about teachers' beliefs, mathematical content knowledge, or pedagogical knowledge or studies that combine just-mentioned phenomena with teachers' teaching practices. In the literature there is also research done solely in K-8 students' cognition for understanding mathematics of students (Olive & Steffe, 2002; Schoenfeld, Smith, & Arcavi, 1993). These studies are related to students' mathematical learning, and they are either unconnected with the school mathematics or the experiences students are having in the classrooms.

However, there is a gap in the literature where classroom instruction, student thinking, and teacher practice are combined to understand the interaction of teaching and learning (Izs‡k, Tillema, & Tun_-Pekkan, 2004): specifically, how students' mathematical learning is shaped through classroom instruction. Cohen and Ball (2001) defined instruction as a function of interactions among teachers, students, and mathematical content, and I will use "instruction" with a similar meaning, but investigate the parts in the definition more, and interactions between the parts for this presentation.

In this interpretive study, I interviewed a pair of sixth-grade, female, white students, who were learning fractions in a classroom that used reform-oriented curricula in a middle school located in a small USA southern town. For a semester when classroom fraction instruction took place, I conducted five interviews each lasting approximately an hour. During interviews, I frequently used classroom video clips taken as a part of a bigger research project (Coordinating Students and Teachers Algebraic Reasoning) to remind interview students some of the classroom instruction. Fraction instruction included number line representations of fractions, comparing fractions and equivalent fractions.

During poster presentation, I will present data and analysis of the interview students' understanding of equivalent fractions on the number line and will discuss how their understandings were supported and shaped by the classroom instruction.

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MULTIMEDIA INSTRUCTIONAL PRESENTATIONS ON LIMIT: EXPERT EVALUATIONS

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Multimedia instructional environments are a new and evolving instructional context that incorporate multiple modalities, e.g. verbal presentations (on-screen text or narration) as well as pictorial presentations (including static and dynamic illustrations). In recent years, the availability of sophisticated technological tools has not only enabled novel presentations of traditional instructional materials but has also shaped the development of novel instructional materials, such as interactive exercises and simulations. In addition, the Internet now allows vast audiences access to these educational materials. As a consequence, online courses and course materials have become publicly available in a number of subjects, including statistics, economics, science, and mathematics.

What factors distinguish different realizations of these courses and determine their effectiveness? The shaping of a framework to guide the systematic evaluation of multimedia instructional messages is on the current agenda of media and educational research (Mioduser, Nachmias, Oren, & Lahav, 1999). With respect to presentational quality, media research has focused on the construction and evaluation of short, context-independent multimedia instructional presentations (Mayer, 1999). With respect to educational quality, interest is focused on longer instructional messages and the extension of cognitive learning theories to multimedia settings (Larreamendy-Joerns, Leinhardt, & Corredor, 2005). However, the overall goal of assessing effectiveness in the larger educational sense remains a challenge (Larreamendy-Joerns & Leinhardt, in press). Our research addresses this challenge by collecting and analyzing the perspectives of experts from relevant disciplines on instructional materials that are part of an online college-level mathematics course. For the context of the instructional message, we chose a foundational yet notoriously problematic mathematical concept that calculus students encounter, namely the limit. Evaluations of two contrasting online course presentations of limit were solicited from experts in mathematics, mathematics education, human-computer interaction, and psychology.

Presentational coordination was a common theme across all experts. The experts focused on the coherence of the instructional exposition both within and between the course components (e.g. exercises, examples, and explanation). In addition, each expert contributed a unique critical perspective: mathematical nuances (mathematician), conceptual connections (mathematics educator), online affordances (human-computer interaction specialist), and associated discourse practices (psychologist). The impact of these findings extends beyond the development of effective online mathematical presentations to the shaping of a genre of critique for online educational materials.

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LEARNING MATHEMATICS IN CENTRAL APPALACHIA: LIFE HISTORIES OF FUTURE ELEMENTARY TEACHERS

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While a perpetual mathematics achievement gap for students in areas of poverty has been clearly documented (Secada, 1992), few studies have explored those results in rural areas (Silver, 2003). Even less research exists for mathematics achievement of students in Central Appalachia, an area of persistent rural poverty, despite the 1983 call for "ethnographic research which examines…internal and external factors shaping school experience" by Keefe, Reck, and Reck (p. 218) who reviewed variables and interactions that comprise the educational experience in Appalachia.

As a native Appalachian with mathematics teaching experience in middle school, I was recruited to help teacher candidates at a Central Appalachia college who were struggling with *Praxis I* – Mathematics, a test required for licensure. In my work with them, I observed that their difficulties appeared to stem from the mathematical content of their public school years.

This poster reflects the research I conducted for my doctoral dissertation. Using an oral history approach, I interviewed three individuals who had all attended public school in Central Appalachia, who were good students in terms of grades, behavior, attendance, and test scores, yet who struggled with the mathematics on the *Praxis I* test. In addition to the interviews, each participant created a graph of their own mathematics self-image for each year of public school.

Life histories for Laura, Faith, and Peyton were constructed. From these narrative data, themes of positive primary grade experiences, shyness, loss of confidence in middle school mathematics, struggles with geometry, choices to take fewer mathematics courses in high school, success in college mathematics, and Appalachian self-image arose. These themes relate to the literature on rural poverty and help to understand the impact of Appalachian culture on mathematics achievement.

Questions were raised regarding effective mathematics teaching for Appalachian students and how it may differ than what is recommended for the mainstream. Future research could include additional life stories from a variety of native Appalachians including those who did not attend college or graduate from high school, as well as those who excelled in mathematics in college and careers, to better understand the various mathematical experiences that students have in school and which were beneficial or detrimental to their learning. Future students could benefit from research that focuses on improving mathematics education in areas of rural poverty, such as Central Appalachia, to narrow the achievement gap and to increase opportunities for success in college and careers.

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