Maple Exercises, Fourier series

These exercises illustrate how *Maple* can be used to analyze compute Fourier series. The easiest form of Fourier series to use is the complex form. The Maple worksheet shows how to compute the Fourier coefficients of a function and plot the Fourier series. Work through the worksheet before trying the problems.

We recall that the Fourier Series for a function on the interval $[0, 1]$ is given by

$$\hat{f} = \sum_{k=-N}^{N} a_k e^{2\pi i k x}$$

where the coefficients a_k are give by

$$a_k = \int_0^1 f(t) e^{-2\pi i k x}\, dx.$$

If $f(x)$ is a real valued function, then $a_k = \overline{a}_{-k}$ and the Fourier series is also a real valued function.

Problems:

(1) Using Maple, compute partial sum for the Fourier series

$$\sum_{k=-N}^{N} a_k e^{2\pi i k x}$$

for the function x^2 on the interval $[0, 1]$ for $N = 2, 3, 4, 5, 6, 10, 20$. In each case plot the series together with the function and notice how closely the series approximates the function.

You might want to write a procedure to compute the real part of the fourier series given a function and a value of N. This will make it easier to do the following problems.

(2) Do the same as in the previous item for the functions
   (a) e^x
   (b) $-x^2 + x + \frac{1}{4}$
   (c) Heaviside$(x - 1/4)$ − Heaviside$(x - 3/4)$

(3) Consider the Fourier series

$$\sum_{j=-N}^{N} c_j \exp(2\pi i j x)$$

with the coefficients

$$c_j = \exp(-2\pi i j x_0).$$

Graph the real part of the Fourier series for $N = 5$ and
   (a) $x_0 = 1/2$
   (b) $x_0 = 1/4$

Experiment with larger values of N.

(4) Consider the 2 variable Fourier series

$$\sum_{k=-N}^{N} \sum_{j=-N}^{N} c_{jk} \exp(2\pi i (jx + ky))$$

with the coefficients

$$c_{jk} = \exp(-2\pi i(jx_0 + ky_0)).$$

Graph the real part of the Fourier series for $N = 5$ and
(a) $(x_0, y_0) = (1/2, 1/2)$
(b) $(x_0, y_0) = (1/4, 1/4)$
Also do a contour plot. Experiment with larger values of N.

(5) Consider the 3 variable Fourier series

$$\sum_{\ell=-N}^{N} \sum_{k=-N}^{N} \sum_{j=-N}^{N} c_{jk} \exp(2\pi i(jx + ky + \ell z))$$

with the coefficients

$$c_{jk} = \exp(-2\pi i(jx_0 + ky_0) + ellz_0).$$

Graph the level curve of real part of the Fourier series equal to 20 for $N = 3$ and
(a) $(x_0, y_0, z_0) = (1/2, 1/2, 1/2)$
(b) $(x_0, y_0, z_0) = (1/4, 1/4, 1/4)$
Use `implicitplot3d` in Maple. Experiment with larger values of N.