Maple Exercises, Rotations

The Maple exercises below illustrate some properties of rotation matrices. First open the Maple worksheet on rotations from the class webpage and work through it. E-mail your worksheet to me when you are finished.

1. Let A be the matrix

$$\begin{pmatrix} 2 & 3 & 1 \\ 2.1 & 2 & 2 \\ 1 & 4 & 0 \end{pmatrix}.$$

Let V be the matrix composed of eigenvectors of A, and let E be diagonal matrix with the corresponding eigenvalues. Use Maple to compute that

$$AV = VE \tag{1}$$

(approximately), i. e., show that the norm of $AV - VE$ is approximately zero.

This should work for any matrix. Try it with another matrix chosen at random.

Equation (1) is a way of writing three eigenvalue equations in matrix form.

2. A rotation matrix is a matrix A such that

$$A'A = I \text{ and } \det A = 1. \tag{2}$$

Let $R_x(\theta)$, $R_y(\theta)$, and $R_z(\theta)$ be rotations an angle of θ around the x,y and z axes respectively and let

$$A = R_y(1.2)R_z(2.1)R_y(5.4).$$

This is the rotation with Euler angles 1.2, 2.1, 5.4. Verify using Maple that the matrix A above satisfies (2) and so it is a rotation matrix. Use Maple to show that the absolute value of every eigenvalue is 1. Show also that Maple returns eigenvectors v such that $v^*v = 1$.

3. Find the axis direction of the rotation matrix with Euler angles $\pi/4$, $\pi/6$, $\pi/12$ by finding the eigenvector corresponding to the eigenvalue 1. Also find the angle of rotation, in degrees, by finding the argument of the complex eigenvector. Use floating point for this. The exact answer is not pretty.

4. Construct a few random rotation matrices by using the random number generator to pick three random Euler angles. Let V be the matrix of eigenvectors. Show in each case $V^*V = I$.