Robust Airfoil Optimization Using Maximum Expected Value and Expected Maximum Value Approaches

In Honor of M.Yousuff Hussaini

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In Honor of M. Yousuff Hussaini

- Ideal Ph.D. / PostDoc Advisor; he guides you through all phases of your Ph.D., then supports you through the rest of your life as you apply for jobs or special opportunities;
- Leader in MANY fields, teaches expertly, and is instrumental in bringing people with common interests together;
- Gives you freedom to make your own discoveries, yet guide you when you need guidance.

THANK YOU DEEPLY, DR. HUSSAINI!

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Robust Airfoil Optimization Using Maximum Expected Value and Expected Maximum Value Approaches

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Deterministic engineering design often leads to unexpected or physically unrealizable results. This is due to the fact that deterministic design is not able to capture the effects of even slight natural fluctuations of parameters. Deterministic transonic shape optimization is no exception: deterministic designs can result in dramatically inferior performance when the actual operating conditions are different from the design conditions used during a deterministic optimization procedure. The goal of this paper is to overcome the off-design performance degradation of deterministic transonic shape optimization by using two different optimization approaches to produce robust designs. Two criteria, the well-known maximum/minimum expected value criterion (MEV) and the alternative expected maximum/minimum value criterion (EMV), are studied and applied to improve an initial RAE 2822 design. It turns out that EMV is much easier to implement than MEV, given a deterministic optimization code, and may provide a promising method for optimizing design shapes under uncertainty.

I. Introduction

Optimization theory, one of the oldest and most mature branches of mathematics, has ubiquitous applications in scientific and engineering disciplines. The main goal of these real-world applications is finding the best choice (the optimum point) that yields the most desirable or satisfactory solution for certain criteria. For example, the optimum point could maximize performance or profitability, or it could minimize risk. The specific example studied in this paper is airfoil shape design, where the objective is to select an airfoil geometry that minimizes the drag coefficient for a given lift.

Traditional gradient-based optimization methods based on finite difference derived gradients [1,2] are not very computationally efficient. With current advances in computational fluid dynamics and modern computers, however, aerodynamic design optimization becomes more feasible than ever, and has been extensively investigated in recent years. Optimization techniques based on adjoint methods [3–6], evolutionary algorithms, or stochastic algorithms [7–10] have been developed and implemented.

The adjoint method is extremely efficient when used with a gradient-based optimization technique, since the necessary gradients are obtained via the solution of the adjoint equations of the governing equations of interest. The computational cost incurred in the calculation of the complete gradient is independent of the number of design variables, and is similar to that of the flow solution. There are a number of partial derivatives that must be evaluated in the adjoint method that do depend on the number of variables, but they are not computationally expensive. The adjoint method was applied in this way to elliptical equations for shape design by Pironneau [11], and was first used in transonic flow by Jameson [12]. Since then, this method has become a popular tool for aerodynamic optimization [3–6]. Unfortunately, there is a drawback associated with the gradient-based methods, and implicitly with the adjoint method: they may get trapped in local minima.

To overcome this, stochastic methods such as genetic algorithms, simulated annealing algorithms, and so on have been applied to aerodynamic shape design. Although these algorithms were inspired by different natural processes, their application to optimization problems shares a common feature: the search for the global optimum through a stochastic process. Genetic algorithms belong to a class of methods called evolutionary algorithms, and they are inspired by the process of natural selection. These evolutionary algorithms have been applied to airfoil shape design by Quagliarella and Cioppa [13], Yamamoto and Inoue [14], and recently by de Sousa and Ramos [8], and Liu [10]. Simulated annealing algorithms, on the other hand, are inspired by the behavior of a collection of atoms immersed in a heat bath subject to a cooling schedule. Aly et al. [15] and Wang et al. [7] have applied annealing algorithms to the design of an optimal aerodynamic shape. The main disadvantage of all these stochastic algorithms is that they usually require a great number of evaluations of the objective function.

Unfortunately, despite the "stochastic" name, all of the aforementioned methods are deterministic in that the conditions or parameters of the problem are all known or predetermined. In reality, the conditions or parameters of the problem under investigation might not be all known beforehand or might be variable due to naturally occurring and irreducible fluctuations. The uncertainty is

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Motivation of Current Research

Deterministic engineering design often leads to unexpected or physically unrealizable results. This is due to the fact that deterministic design is not able to capture the effects of even slight natural fluctuations of parameters. **Deterministic transonic shape optimization** is no exception—deterministic designs can result in dramatically inferior performance when the actual operating conditions are different from the design conditions used during a deterministic optimization procedure.

What can be done to overcome the shortcomings of the deterministic design (deterministic transonic shape optimization)?

Goals of Current Research

The goals of this research are to

- overcome the off-design performance degradation of deterministic transonic shape optimization to produce **robust designs**;
- consider **innovative ways** to achieve the robustness of the designs;
- **improve** an initial Royal Aircraft Establishment (RAE) 2822 design.

Problem Formulation: Airfoil Shape Optimization Under Uncertainty in the Operating Conditions

Our goal is to minimize the drag coefficient C_D subject to a prescribed lift coefficient C_L^*, and some other geometrical constraints h_i, i.e.

$$\min_{\mathbf{d}\in\mathbf{D}} C_D\left(\mathbf{d}, M\right), \text{ over a range of M}$$
$$\text{subject to } C_L = C_L^* \text{ and } h_i\left(\mathbf{d}, M\right) \geq 0, \text{ for } i = 1, 2, ..., n \tag{1}$$

The desired shape **d** is an element of a design space **D**, and the random Mach number M is defined on a given probability space $(\Omega, \mathcal{F}, P)$ with probability density function $f(M)$.

We will use the compressible Euler equations as the mathematical model of the governing equations of the flow.

Deterministic Airfoil Shape Optimization

- The optimization problem (1) carried out over a single value of the Mach number M is deterministic, as it does not take into account any probabilistic properties of the random variable M.
- The deterministic optimum shape can be determined using control theory approach.
- The governing equations of the flow field are introduced as a constraint in such a way that the final expression for the gradient of the objective function does not require reevaluation of the flow field.
- To achieve this purpose, a Lagrange multiplier (or co-state variable) is introduced to satisfy the so-called adjoint equation.

Deterministic Design Procedure

1. Initialize the deterministic parameters involved in the optimization procedure, parameterize the configuration of interest using a set of design parameters, and define the initial shape;
2. Solve the flow equations for the flow variables density ρ, velocity components u_1, u_2, u_3, and pressure p;
3. Solve the adjoint equations for the co-state variables subject to appropriate boundary conditions;
4. Evaluate the gradients and update the aerodynamic shape based on the direction of steepest descent (for instance);
5. Return to step #2 until an optimum configuration is attained.

Shortcomings of the Deterministic Design

Practical examples indicate that the deterministic optimization approach described above can result in **dramatically inferior performance when the actual operating conditions are different from the design values used for the optimization**. Therefore, practical robust designs need to be considered for achieving consistent drag reduction **over a given Mach number range** (first objective of the present research work).

Robust Design Approaches to Airfoil Shape Optimization Under Uncertainty

Robust Design = Optimization problem (1) (minimization of the random loss function $C_D(\mathbf{d}, M)$).
The stochastic minimizaton can be reformulated in the following ways/according to the following objectives:

1. Find the design that minimizes the variance of the objective function;
2. Find the worst-case performance (MiniMax strategy);
3. Find the design that minimizes the expectation of the objective function (MEV);
4. Find the design that improves the performance over a given range of uncertain parameters compared to the MEV design (second objective of the present research work).

Maximum/Minimum Expected Value (MEV) Criterion

The MEV optimum shape defined in aerodynamics by Huyse, Padula, Lewis, and Li:

$$\mathbf{d}_{MEV} = \arg\min_{\mathbf{d}\in\mathbf{D}} E\left(C_D\left(\mathbf{d}, M\right)\right) = \arg\min_{\mathbf{d}\in\mathbf{D}} \int_{\Omega} C_D\left(\mathbf{d}, M\right) f\left(M\right) dM \tag{2}$$

In order to approximate the right hand side of (2), we let $M_1, M_2, ..., M_n$ be a random sample of size n of the random variable M with probability density function f. A Monte Carlo-type estimator defined by

$$\mathbf{d}^n_{MEV} = \arg\min_{\mathbf{d}\in\mathbf{D}} \left[\frac{1}{n}\sum_{k=1}^{n} C_D\left(\mathbf{d}, M_k\right)\right]$$

converges to the optimum $\mathbf{d}_{MEV}$, as $n \to \infty$.

MEV Design Procedure

1. Initialize the deterministic parameters, parameterize the configuration of interest using a set of design variables, and define the initial shape;
2. Sample all the values of the random variable M according to its probability distribution function $f(M)$;
3. For every value of the random variable M, solve the flow equations for the flow variables ρ, u_1, u_2, u_3, p;
4. For every value of the random variable M and the current airfoil shape, solve the adjoint equations for the co-state variables ψ subject to appropriate boundary conditions;
5. Evaluate the gradients G for every value of the random variable M, average all the gradients and update the aerodynamic shape based on the averaged direction of steepest descent (for instance);
6. Return to step #3 and repeat until an MEV optimum configuration is attained.

Motivation of Expected Minimum / Maximum Value (EMV) Criterion

- Recall the Minimum / Maximum Expected Value (MEV) Criterion is

$$\mathbf{d}_{MEV} = \arg\min_{\mathbf{d}\in\mathbf{D}} E\left(C_D\left(\mathbf{d}, M\right)\right)$$

- Question: What if we commute the expectation and minimization operators?

$$\mathbf{d}_{EMV} = E\left(\arg\min_{\mathbf{d}\in\mathbf{D}} C_D\left(\mathbf{d}, M\right)\right)$$

- We call this strategy Expected Maximum/Minimum Value (EMV) criterion and we would like to know how it compares to MEV.

Properties of Expected Minimum / Maximum Value Criterion

It can be proved that:

- If the objective function $C_D(\mathbf{d}, M)$ to be minimized is continuos and $\mathbf{d}_{MEV}$ and $\mathbf{d}_{EMV}$ are well-defined, then

$$E\left(\min_{\mathbf{d}\in D} C_D\left(\mathbf{d}, M\right)\right) \leq \min_{\mathbf{d}\in D} E\left(C_D\left(\mathbf{d}, M\right)\right)$$

- Croicu and Hussaini have proven that in some illustrative cases, the EMV method provides a higher probability of lower objective function than the MEV approach, i.e.

$$P\left(C_D\left(\mathbf{d}_{EMV}, M\right) \leq C_D\left(\mathbf{d}_{MEV}, M\right)\right) \geq 50\%.$$

Expected Maximum/Minimum Value (EMV) Criterion

$$\mathbf{d}_{EMV} = E\left(\arg\min_{\mathbf{d}\in\mathbf{D}} C_D(\mathbf{d}, M)\right) = \int_{\Omega} \arg\min_{\mathbf{d}\in\mathbf{D}} C_D(\mathbf{d}, M)\, f(M)\, dM \tag{3}$$

In order to approximate the right hand side of (3), we let $M_1, M_2, ..., M_n$ be a random sample of size n of the random variable M with probability density function f. For each sample $M_k, k = 1, 2, ..., n$ value, the optimization problem (1) yields a solution $\mathbf{d}(M_k), k = 1, 2, ..., n$. The averaged optimal shape:

$$\mathbf{d}_{EMV}^{n} = \frac{1}{n}\sum_{k=1}^{n} \mathbf{d}(M_k) = \frac{1}{n}\sum_{k=1}^{n} \arg\min_{\mathbf{d}\in\mathbf{D}} C_D(\mathbf{d}, M_k)$$

If the right hand side of (3) is finite, then by the Strong Law of Large Numbers, the Monte Carlo-type estimator $\mathbf{d}_{EMV}^{n}$ converges

$$\mathbf{d}_{EMV}^{n} \to \mathbf{d}_{EMV} \text{ as } n \to \infty.$$

EMV Design Procedure

1. Initialize the deterministic parameters, and parameterize the configuration of interest using a set of design variables;
2. Sample a value of the random parameter M, according to its probability distribution function $f(M)$, and define the chosen initial shape;
3. Solve the flow equations for the flow variables ρ, u_1, u_2, u_3, p;
4. Solve the adjoint equations for the co-state variables ψ subject to appropriate boundary conditions;
5. Evaluate the gradients G and update the aerodynamic shape based on the direction of steepest descent (for instance);
6. Return to step #3 until an optimum configuration is attained;
7. Return to step #2 and repeat until a desired number of sample points are analyzed;
8. Average all the optimum configurations obtained in step #6 to determine the EMV optimum design.

Computer Implementation (contd.)

- The design of a transonic airfoil that performs well over a range of different Mach numbers is investigated using the EMV and MEV approaches and the CFD code SYN83 (Anthony Jameson).
- SYN83 is an implementation of a gradient-based optimization technique in which the control variable—the airfoil shape **d**—is parametrized using a set of 65 design nodes $\{(x_i, y_i), i = 1, 2, ..., 65\}$

Computer Implementation (contd.)

- The drag coefficient $C_D(\mathbf{d}, M)$ is the objective function to be minimized.
- The gradient information is obtained via the adjoint equation, and this adjoint equation is used to calculate the sensitivity derivatives of the cost function with respect to the design variables, in order to get a direction of improvement. The flow is calculated using the steady-state inviscid Euler equations.
- The initial shape is the well-known RAE 2822 profile with an imposed lift coefficient $C_L = 0.5$. Because this shape is not suitable for the transonic regime, substantial improvements are to be expected.

Computer Implementation (contd.)

- We assume the following three distributions of the Mach number: Uniform Distribution $M \sim \mathcal{U}[0.7, 0.8]$, Gaussian Distribution $M \sim \mathcal{N}\left(0.75, (0.02)^2\right)$, and Gaussian Distribution $M \sim \mathcal{N}\left(0.775, (0.01)^2\right)$.
- Different distributions are chosen with the aim of emphasizing the importance of accurately quantifying the probability distribution function (PDF) of the Mach range.
- For practical problems, the PDF is likely to be very different from the distributions used in our numerical simulations. The actual PDF can be generated if appropriate historical flight data is available.

Computer Implementation (contd.)

- For both stochastic optimization procedures (EMV or MEV) and for each distribution, we choose 1000 samples of the Mach number.
- An optimal shape is found using the EMV and the MEV methods, as well as an Single Point Optimization (SPO) at the mean of the Mach number distributions.

Results

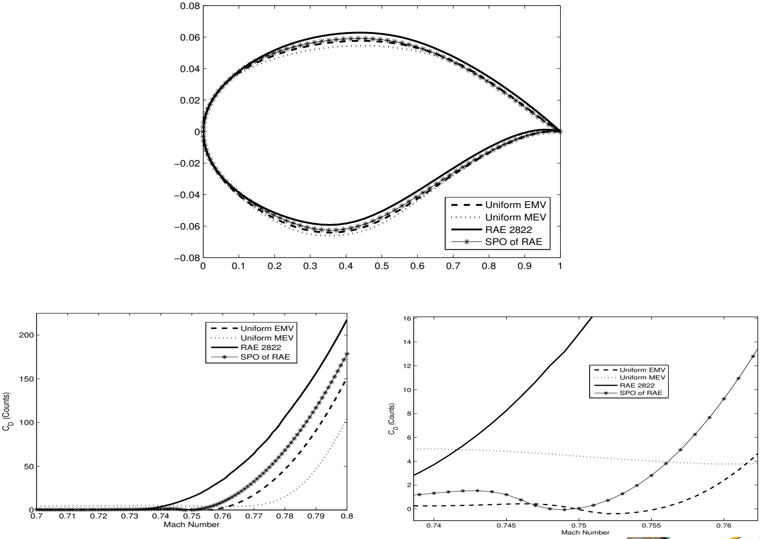

Figure: Shape and Drag Coefficient for Uniform $M \sim \mathcal{U}[0.7, 0.8]$

Results (contd.)

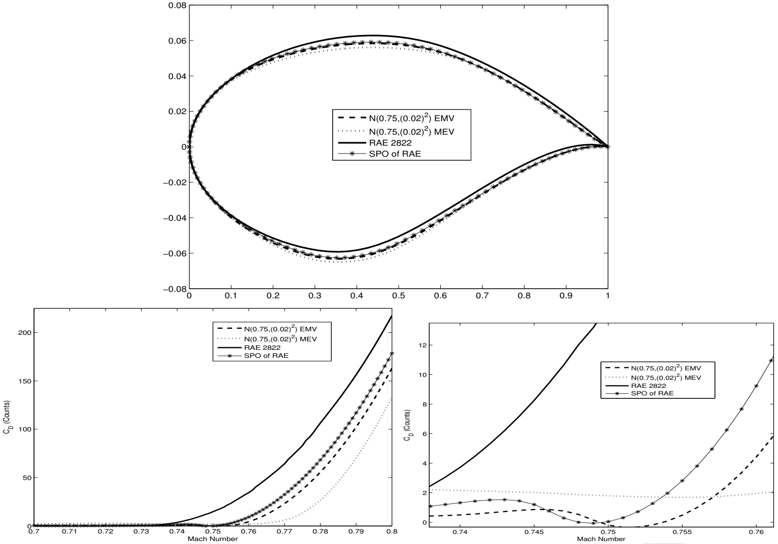

Figure: Shape and Drag Coefficient for Gaussian $M \sim \mathcal{N}\left(0.75, (0.02)^2\right)$

Results (contd.)

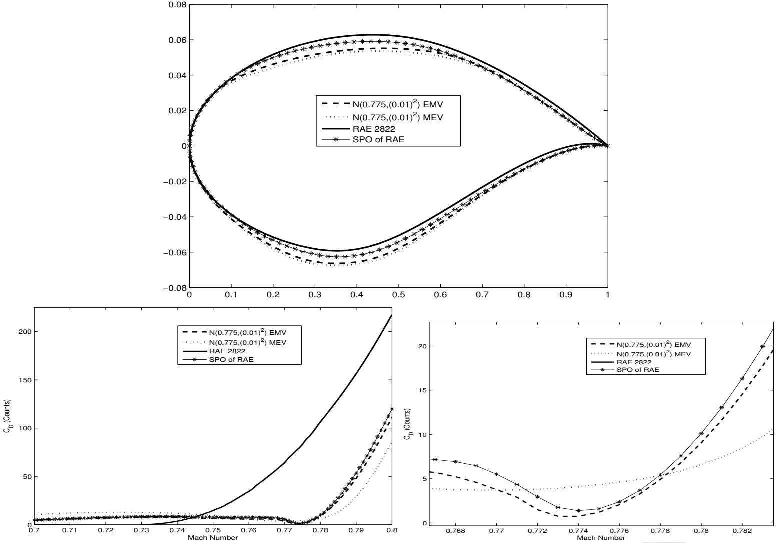

Figure: Shape and Drag Coefficient for Gaussian $M \sim \mathcal{N}\left(0.775, (0.01)^2\right)$

Concluding Remarks

- We focused on obtaining a robust optimal design starting from an RAE 2822 airfoil with Mach number as an uncertain parameter.
- Two different stochastic approaches—EMV and MEV—have been investigated and compared to the deterministic SPO.
- As expected, SPO degrades rapidly away from the design point and does not provide robust results.

Concluding Remarks (contd.)

- The EMV strategy is easier to implement if a deterministic optimization code is available, provides lower drag for low speeds, lower drag for more than 50% of the Mach range, and similar performance to the SPO profile at the mean Mach design point.
- MEV profile exhibits lower drag at high speeds and the lowest expected drag.
- Therefore, the appropriate choice between the EMV and MEV strategies would depend on some other considerations: the magnitude of the shock-waves, the most plausible speed regime and speed distribution, the ease of computational implementations, etc.

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THANK YOU!

QUESTIONS?