

## Algebraic geometry : HW 6 solution

1. Prove that if  $B$  is a domain, then  $B$  is the intersection inside the quotient field of  $B$  of the localizations of  $B$  at maximal ideals of  $B$ .

*Solution:* Certainly  $B$  is contained in the intersection inside the quotient field of  $B$  of the localizations of  $B$  at maximal ideals of  $B$ . Conversely suppose  $f/g$  is an element of the quotient field of  $B$  that is contained in  $B_{\mathfrak{m}}$  for all maximal ideals  $\mathfrak{m}$  of  $B$ . Then  $g \notin \mathfrak{m}$  for all maximal ideals  $\mathfrak{m}$  of  $B$ , and so  $g$  is a unit in  $B$  (if it is not, then  $g$  is contained in the maximal ideal containing the ideal generated by  $g$ ). Thus  $f/g \in B$ , which proves the reverse inclusion.