Abstract

Gromov hyperbolic spaces are a generalization of the ordinary hyperbolic spaces of Riemannian geometry. If a group $\Gamma$ acts on such a space $X$, the action can be extended to a natural compactification $\overline{X}$ of $X$. Usually, the action on the boundary $\partial X = \overline{X} \setminus X$ is not well behaved. This means that on a part of the boundary called the limit set $\Lambda$ of $\Gamma$, the orbits of the action are dense. Then, the quotient space $\Lambda/\Gamma$ carries the trivial topology. Thus, for such actions, the techniques of ordinary topology and geometry are not sufficient.

The theory of operator algebras and noncommutative geometry does provide a framework to approach this kind of problems. In this talk I will explain some of the ideas of noncommutative geometry and how they might be applied to boundary actions.

Examples of Gromov hyperbolic spaces include the complex upper half plane, $p$-adic vector spaces and trees. Via these examples, the techniques of noncommutative geometry are made available for application to arithmetic problems.