GRV II : HW 7

1. Show that the center of a direct product is the direct product of the centers:

$$Z(G_1 \times \cdots \times G_n) = Z(G_1) \times \cdots \times Z(G_n).$$

2. Let $G_1, \ldots, G_n$ be groups and let π be a fixed element of S_n. Prove that the map

$$\phi_\pi : G_1 \times \cdots \times G_n \to G_{\pi^{-1}(1)} \times \cdots \times G_{\pi^{-1}(n)}$$

defined by

$$\phi_\pi(g_1, \ldots, g_n) = g_{\pi^{-1}(1)} \times \cdots \times g_{\pi^{-1}(n)}$$

is an isomorphism (so that changing the order of the factors in a direct product does not change the isomorphism type).

3. Give the number of nonisomorphic abelian groups of order 100.

4. Give the lists of elementary divisors for all abelian groups of order 270.

5. Let G be a finite abelian group of type $(n_1, n_2, \ldots, n_t)$. Prove that G contains an element of order m if and only if $m|n_1$. Deduce that G is of exponent n_1.

6. Show that the following are composition series for the group D_8:

$$1 \leq \langle s \rangle \leq \langle s, r^2 \rangle \leq D_8$$

$$1 \leq \langle r^2 \rangle \leq \langle r \rangle \leq \langle D_8 \rangle.$$

Find their composition factors and show directly that they have the same three composition factors (as the Jordan-Holder theorem predicts).