

## GRV III : HW 5

I have not listed below the optional problems 8 and 15 from §14.2.

1. Determine the minimal polynomial over  $\mathbf{Q}$  for the element  $\sqrt{2} + \sqrt{5}$ .
2. Determine all the distinct subfields of  $\mathbf{Q}(\sqrt{2}, \sqrt{5})$ . Be sure to justify your answer as much as possible.
3. Let  $K/F$  be a finite Galois extension of fields with Galois group  $G$ . Let  $E_1$  and  $E_2$  be subextensions and let  $H_1$  and  $H_2$  be the corresponding subgroups of  $G$  under the Galois correspondence. Show that under this correspondence,  $E_1 \cap E_2$  corresponds to  $\langle H_1, H_2 \rangle$ , the subgroup of  $G$  generated by  $H_1$  and  $H_2$ .