The Bicycle Race

Carol is a local bicycle racing star and today she is in the race of her life. Moving at a constant velocity k meters per second, she passes a refreshment station. At that instant ($t = 0$ seconds), her support car starts from the refreshment station to accelerate after her, beginning from a dead stop. Suppose the distance travelled by Carol in t seconds is given by the expression kt and the distance travelled by the support car is given by the function $\frac{1}{3}(10t^2 - t^3)$, where distance is measured in meters. This latter function is carefully calculated by her crew so that at the instant the car catches up to the racer, they will match speeds. A crew member will hand Carol a cold drink and the car will immediately fall behind.

(a) How fast is Carol travelling? And how long does it take the support car to catch her?

(b) Suppose that Carol is riding at a constant velocity k, which may be different than the value found in (a). Find an expression for the times when the car and the bike meet which gives these times as a function of her velocity k. How many times would the car and the bike meet if Carol were going faster than the velocity in (a)? Slower?

(c) Consider a pair of axes with time measured horizontally and distance vertically. Draw graphs that depict the distance travelled by Carol and by the car plotted on the same axes for the original problem (parts (a) and (b)) and for the questions of part (c). You should have three graphs: one for the bike's velocity found in part (a), one for a faster bike, and one for a slower bike. If Carol had been going any faster or slower than the velocity you found in (a), passing the drink would not have been so easy. Why? Justify your answer.

(d) Prove that if a cubic polynomial $P(x)$ has a double root at $x = a$, then $P'(a) = 0$.

From the Course Syllabus

PROJECT: You will work on the project in groups of 1–4 students. This project will be a substantial assignment, giving you a chance to earn part of your grade in an environment which simulates the so-called "real world" better than does an in-class exam. It will also give your instructor a chance to base part of your grade on your best work, produced in a setting where time should not be a factor (assuming you start on your project as soon as it is assigned). The results of your work on your project will be presented in a report (one report per group). Each member will also submit a "group evaluation" giving their impression of the relative contribution of each member to the group's effort. These evaluations are due with the project. It is not guaranteed that each member of the group will receive the same grade. **The reports will be graded not only on their mathematical content but also on the quality of the presentation: clarity, neatness, and proper grammar are also important. Both reports and group evaluations must be typed.** The project will be assigned on Thursday, October 19 and due on Thursday, November 2.