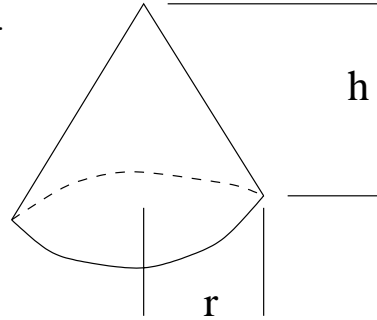


Related Rates

0. A pile of sand in the shape of a cone whose radius is twice its height is growing at a rate of 5 cubic meters per second. How fast is its height increasing when the radius is 20 meters?

These problems are called related rates and basically are all solved the same way. Here is a list of the usual steps:

1. Draw a picture or a figure representing the problem.



2. Label the knowns and unknowns, state what is given and what is wanted.

We know $r = 2h$, $\frac{dV}{dt} = 5m^3/s$ and we want $\frac{dh}{dt}$ when $r = 20m$.

3. Write an equation relating the variables which is true for all time t .

$$V = \frac{1}{3}\pi r^2 h = \frac{4}{3}\pi h^3$$

4. Differentiate the equation in #3 with respect to time.

$$\frac{dV}{dt} = 4\pi h^2 \frac{dh}{dt}$$

5. Use the equation in #3 to find the value of the unknown variables at the given time.

Since $r = 2h$, when $r = 20m$, $h = 10m$

6. Substitute everything into the equation in #4 to find the desired rate.

$$5 = 4\pi 10^2 \frac{dh}{dt} \text{ or } \frac{dh}{dt} = 1/(80\pi) \text{ m/s.}$$

Related rate problems in the text: p 223 32, 33, 34, 36; p 235 7, 15, 16, 17, 18, 19. Here are more:

1. A 6 foot woman is 20 feet away from a 30 foot high lamp post. How fast is her shadow increasing if she is walking away from the lamp at 2 feet per second?
2. A 6 foot woman is 20 feet away from a 30 foot high lamp post. How fast is her shadow decreasing if she is walking toward the lamp at 3 feet per second?
3. A 6 foot woman is 20 feet away from a 30 foot high lamp post. How fast is her shadow increasing if the light is falling down at 2 feet per second?
4. A 6 foot woman is 20 feet away from a 30 foot high lamp post. How fast is her shadow decreasing if the light is rising up at 3 feet per second?
5. A floor lamp is 30 feet away from a high wall and directly between the lamp and the wall is a 6 foot woman 20 feet from the wall. How fast is her shadow changing if she is walking away from the lamp at 2 feet per second?
6. A floor lamp is 30 feet away from a high wall and directly between the lamp and the wall is a 6 foot woman 20 feet from the wall. How fast is her shadow changing if she is walking toward the lamp at 3 feet per second?
7. A floor lamp is 30 feet away from a high wall and directly between the lamp and the wall is a 6 foot woman 20 feet from the wall. How fast is her shadow changing if the lamp is moving 3 feet per second toward the wall?
8. A floor lamp is 30 feet away from a high wall and directly between the lamp and the wall is a 6 foot woman 20 feet from the wall. How fast is her shadow changing if the lamp is moving 2 feet per second away from the wall?