MAC 2311 Calculus 1 **Test 2** 17 Oct 2000

Show **ALL** work for credit; be neat; and use only **ONE** side of each page of paper. Do **NOT** write on this page. Calculators can be used for graphing and calculating only. Give exact answers when possible.

1. Use your calculator to find both the left sum for n = 10 and the numerical integration answer for $\int_0^2 x(2-x)e^{x^2}\,dx$

2. Find an equation for the tangent line to $f(x) = \sqrt{x}$ at $x = 4$. Plot $f(x)$ and this tangent line.

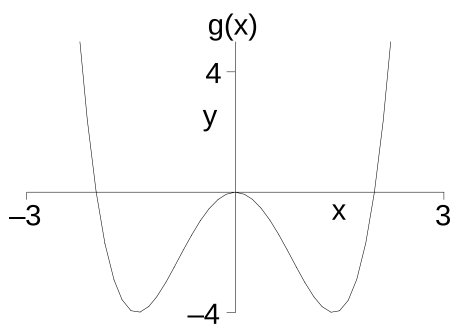
3. The good Doctor decides to run a marathon. His chairman, Professor Sumners, rides behind him on a bicycle and clocks his speed every 3 minutes. Our runner is out of shape and quickly gives up after only 15 minutes. The chairman's data is collected below.

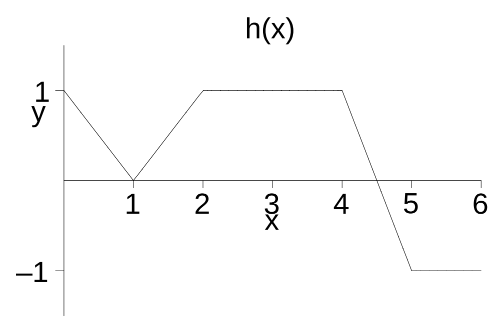
t(minutes)	0	3	6	9	12	15
$v(t)$(m/sec)	2.5	2.0	1.5	1.5	1.0	0.0

(a) Give upper and lower estimates for the distance the good Doctor ran during this quarter hour. (Assume that his speed is always decreasing.)
(b) How often would Professor Sumners have needed to measure the Doctor's speed (or lack thereof) in order to find lower and upper estimates within 75 meters of the actual distance that he ran?

4. Suppose $P = f(t)$ is the population of Mexico in millions, where t is the number of years since 1980. Explain the meaning of the statements:
(a) $f'(6) = 2$ (b) $f^{-1}(95.5) = 16$ (c) $(f^{-1})'(95.5) = 0.46$

5. Sketch the graph of the derivative to the function $g(x)$ in the graph below. (You might want to trace $g(x)$ onto your answer sheet.)





6. The graph above plots $h(x)$. If $H' = h$ and $H(0) = 2$ find $H(b)$ for $b = 1, 2, 3, 4, 5$, and 6.

7. A continuous function, f, defined for all x has the following properties:
- f is increasing
- f is concave down
- $f(5) = 2$
- $f'(5) = \frac{1}{2}$

(a) Sketch a possible graph for f.
(b) How many zero's does f have?
(c) What can you say about the location of the zero's?
(d) What is $\lim_{x\to-\infty} f(x)$?
(e) Is is possible that $f'(1) = 1$? Or $f'(1) = \frac{1}{4}$?

8. Find the derivative of $f(x) = 1/x^2$ algebraically (i.e. using the limit definition).

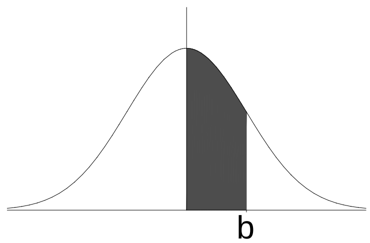
There is more test on the other side.

Welcome to test two side two.

9. The function for the *standard normal distribution*, (sometimes called the bell-shaped curve) which is often used in statistics, has the formula

$$\frac{1}{\sqrt{2\pi}}e^{-x^2/2}$$

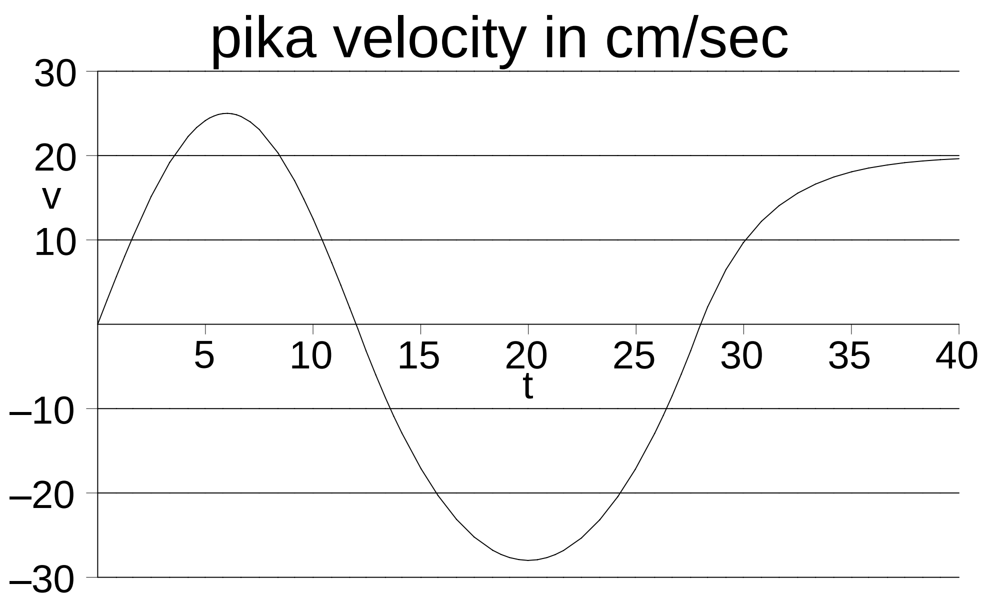
and the graph shown below. Statistics books usually contain tables such as the one below, showing only the the area of the curve from 0 to b, for different values of b.



b	$\frac{1}{\sqrt{2\pi}}\int_0^b e^{-x^2/2}dx$
1	0.3413
2	0.4772
3	0.4987
4	0.5000

Use the informative in the table and the symmetry of the standard normal curve about the y-axis to find:

$(a)\quad \frac{1}{\sqrt{2\pi}}\int_1^2 e^{-x^2/2}dx$ $\qquad (b)\quad \frac{1}{\sqrt{2\pi}}\int_{-2}^3 e^{-x^2/2}dx$



10. A pika moves back and forth in a tunnel attracted to bits of sunflower seeds alternately introduced to and removed from the ends (right and left) of the tunnel. The graph of the pika's velocity, v in cm/sec, is given in the graph above vs the time, t in seconds, with the positive velocity corresponding to the motion toward the right end. Assuming the pika starts ($t = 0$) at the center of the tunnel, use the graph to estimate the time(s) at which

(a) The pika changes direction.
(b) The pika is moving most rapidly to the right; to the left.
(c) The pika is farthest to the right of center; farthest to the left.
(d) The pika's speed (i.e. the magnitude of its velocity) is decreasing.
(e) The pika is at the center of the tunnel.