MAC 2311 Calculus 1 **Test 3** 28 Nov 2000

Show **ALL** work for credit; be neat; and use only **ONE** side of each page of paper. Do **NOT** write on this page. Calculators can be used for graphing and calculating only. Give exact answers when possible.

1. Do the indicated operation.
   a. Find y' if $y = \cos x + \ln x + \arctan x + 2^x + \pi^2$
   b. Find $\int \cos x + e^x + x^{100} + \sec^2 x + \pi \quad dx$.

2. Use L'Hopital's rule to find

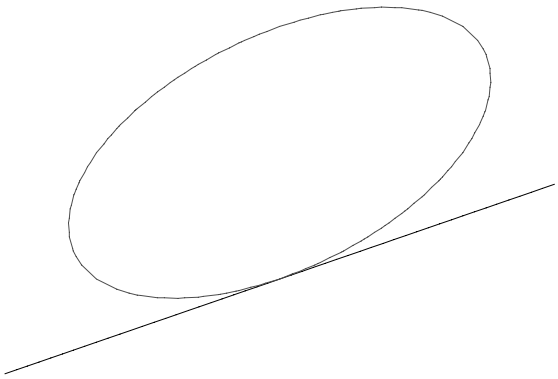
$$\lim_{x \to \infty} \frac{e^x}{x^2}$$

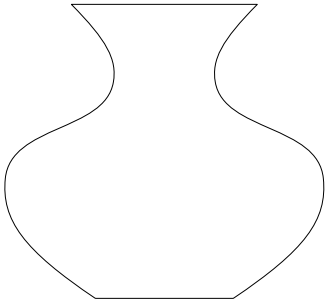
3. Find the (global) minimum and maximum values of the function $f(x) = x^3 - 6x^2 + 9x + 10$ for $2 \leq x \leq 4$.

4. Let $f(x)$ and $g(x)$ be two functions. Values of $f(x)$, $f'(x)$, $g(x)$ and $g'(x)$ for $x = 0, 1$ and 2 are given in the table to the below right. Use the information in the table to find:
   a. $G'(1)$ if $G(x) = f(x)/g(x)$
   b. $H'(0)$ if $H(x) = e^{f(x)} + \pi x$
   c. $J'(1)$ if $J(x) = [f(x)]^2$
   d. $K'(0)$ if $K(x) = f(g(x))$

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	1	-1	2	3
1	-1	2	4	0
2	7	3	11	0.5

5. Consider the equation $x^2 - 2x + y^2 - 2y - xy + 3 = 0$ (see graph below left).
   a. Find dy/dx by implicit differentiation.
   b. Find the equation of the tangent line to the curve when $x = 2$ and $y = 1$ (see graph below left).





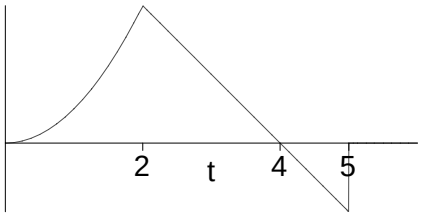
6. Starting at time $t = 0$, water is poured **at a constant rate** into an empty vase (pictured above right). It takes ten seconds for the vase to be filled completely to the top. Let $h = f(t)$ be the depth of the water at time t. Sketch a graph of $h = f(t)$, label the regions where the function is concave up and where it is concave down.

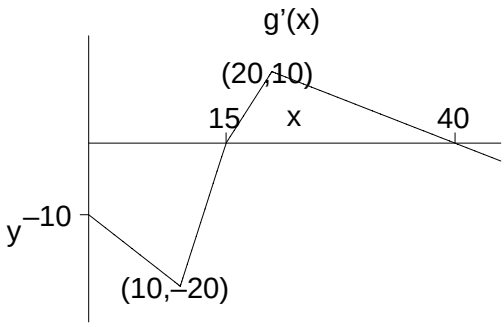
There is more test on the other side.

7. A young girl who aspires to be a rocket scientist launches a model rocket from the ground at time $t = 0$. The rocket travels straight up in the air, and the graph below (left) shows the upward velocity of the rocket as a function of time. The velocity is zero for $t > 5$.

a. Sketch a graph of the acceleration of the rocket as a function of time.

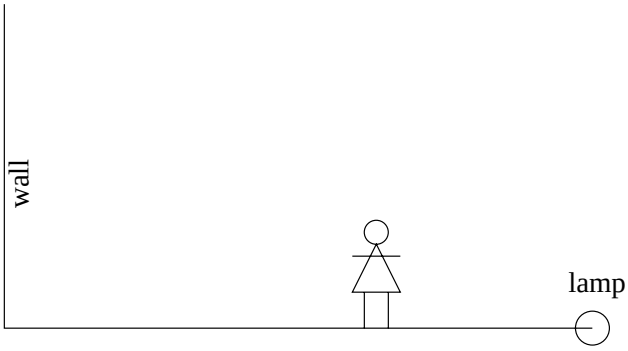
bcd. Sketch a graph of the height of the rocket as a function of time. (Carefully show concavity, label local extrema and points of inflection.)

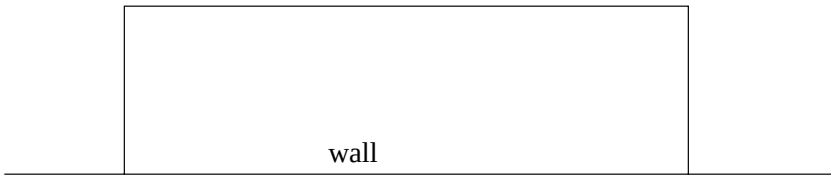




8. The graph above (right) plots the derivative $g'(x)$ of the function $g(x)$. It is given that $g(0) = 50$. Sketch the graph for $g(x)$, showing all critical points and inflection points of g and giving their coordinates.

9. A floor lamp is 30 feet away from a high wall and directly between the lamp and the wall is a 6 foot women 20 feet from the wall (see below left). How fast is her shadow changing if she is walking away from the lamp at 2 feet per second?





10. A landscape architect plans to enclose a 5000 square foot rectanglar region against a long straight wall (see picture above right). The fencing need for the three non-wall sides costs \$16 a foot, while the treatment for the part of wall in the region costs \$9 a foot. Find the minimum total cost.