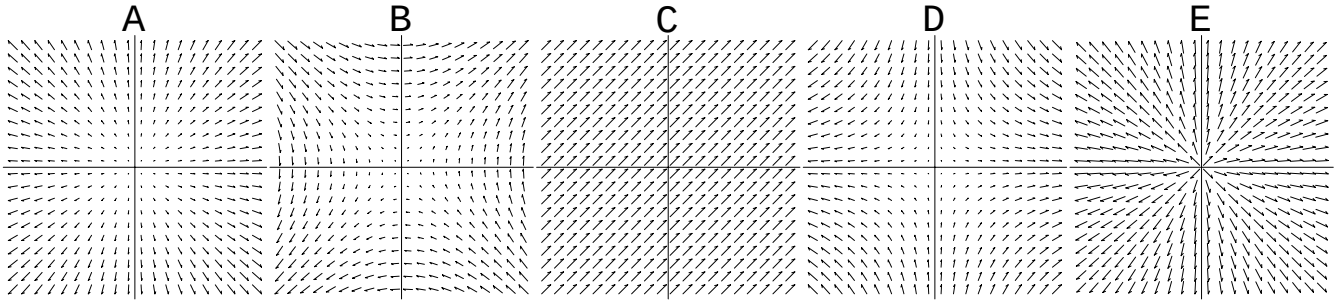
MAC 2313 Calculus 3 **Test 2** 24 Oct 2000

Show **ALL** work for credit; be neat; and use only **ONE** side of each page of paper. Do **NOT** write on this page. Calculators can be used for graphing and calculating only. Give exact answers when possible.

1. Find the equation of the tangent plane to $z = xe^y$ at $(x, y) = (2, 1)$

2. Suppose that $F(x, y, z) = x^2 + y^4 + x^2z^2$ gives the concentration of salt in a fluid at the point (x, y, z) and you are at the point $(-1, 1, 1)$.
(a) In which direction (unit vector) should you move if you want the concentration to increase the fastest?
(b) Suppose you start to move in the direction you found in part (a) at a speed of 4 units/sec. How fast is the concentration changing?

3. The following are maple plots of the gradients of functions $xy, x^2 + y^2, \sqrt{x^2 + y^2}, x + y$, and $x^2 - y^2$ over the range $x = -3..3$ and $y = -3..3$. Match the gradient to the function.



4. Use the Chain Rule to find $\partial z/\partial$ u and $\partial z/\partial v$ when $z = (x + y)e^x, x = \ln u + \sin v$ and $y = v^2 - u^2$.

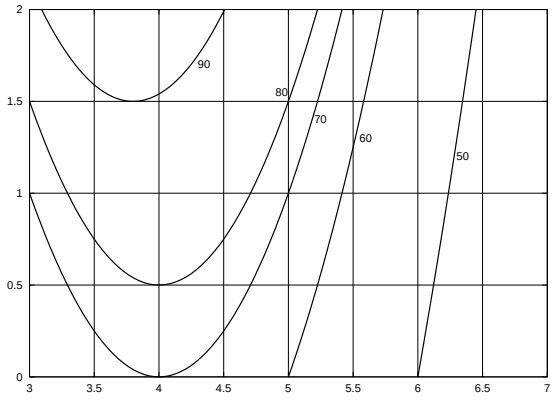
5. Find the local extrema of the function $f(x, y) = 8y^3 + 12x^2 - 24xy$. [Hint: there are two critical points.]

6. The function $f(x, y)$ has local maximums at $(1, 0)$ and $(-1, 0)$, local minimums at $(0, 1)$ and $(0, -1)$ and one saddle point at the orgin.
(a) Sketch a possible contour graph for f.
(b) Sketch a possible graph for ∇f.

7. Use Lagrange Multipliers to find the maximum and minimum **VALUES** of $6x+8y$ on the circle $x^2+y^2 = 25$.

8. The function f is given by the table below left and the function g is given by the contour plot below right.
(a) Give your best estimates of $f_x(5, 1)$ and $f_y(5, 1)$.
(b) Give your best estimates of $g_x(5, 1)$ and $g_y(5, 1)$.

$f(x, y)$	$x = 3$	$x = 4$	$x = 5$	$x = 6$
$y = 0$	0.6	0.0	0.6	1.3
$y = 1$	1.0	0.8	1.0	2.0
$y = 2$	3.0	2.6	3.0	4.0
$y = 3$	5.2	4.7	5.2	7.7



9. Maple questions:

(a) Why do we use "scaling=constrained" in maple plots with "arrows" or gradients?

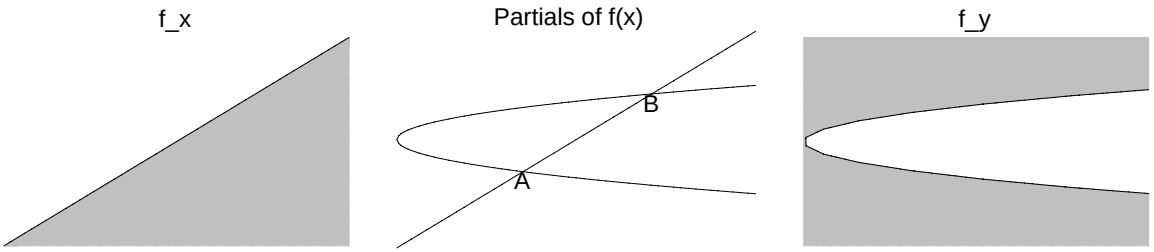
(b) You carefully type "contourplot(x^2+y^2,x=0..1,y=0..1);" into Maple and Maple annoyingly just echos "contourplot(x^2+y^2,x=0..1,y=0..1);", what is wrong and how do you fix it?

(c) Fix the error in the Maple command "plot(exp^x,x=0..1);"

You wanted to plot $x^2 + y^2 - z^2 = 1$ but told Maple "implicitplot3d(x^2+y^2-z^2,x=-3..3,y=-3..3,z=-3..3,numpoints=10000);".

(d) What kind of graph did Maple draw?

(e) Fix the command so it draws the correct graph.



10. The graphs above are graphs of the partial derivatives of the function $f(x, y)$. The leftmost graph is a "contour" graph of f_x. The graph is gray where $f_x > 0$, white where $f_x < 0$ and the black line is where $f_x = 0$. The rightmost graph, is a similar "contour" graph for f_y. The middle graph plots the two equations $f_x = 0, f_y = 0$ showing the two critical points A and B. (Finding the sign means determining if the quantity is positive, negative or zero.)

(a) Find the sign of f_{xx} at both the points A and B.

(b) Find the sign of f_{yy} at both the points A and B.

(c) Suppose f_x is a linear function, find the sign of f_{xy} at both the points A and B.

(d) Can f have a local max at A? Can f have a local min at A? Can A be a saddle point for f? Explain.

(e) Can f have a local max at B? Can f have a local min at B? Can B be a saddle point for f? Explain.