4.8#12 Since $f'(x) = 3\cos 3x$, so $f'(0) = 3$ is the slope at $x = 0$. Since $g'(x) = 5$, so $g'(0) = 5$ is the slope at $x = 0$. Thus by L'Hopital's rule

$$lim_{x\to 0}\frac{\sin 3x}{5x} = lim_{x\to 0}\frac{3}{5} = \frac{3}{5}$$

7.5#19 . Let $\Delta x = (b-a)/n$ and $x_0 = a, x_1 = x_0 + \Delta x, \ldots x_i = x_0 + i\Delta x \ldots x_n = x_0 + n\Delta x = b$. Then

$$RIGHT(n) = \sum_{i=1}^{n} f(x_i)\Delta x \qquad LEFT(n) = \sum_{i=0}^{n-1} f(x_i)\Delta x.$$

Note that these are the same terms but for the last term of $RIGHT(n)$ and the first term of $LEFT(n)$ so $RIGHT(n) - LEFT(n) = f(x_n)\Delta x - f(x_0)\Delta x = f(b)\Delta x - f(a)\Delta x$. Adding $LEFT(n)$ to both sides produces the desired equation.

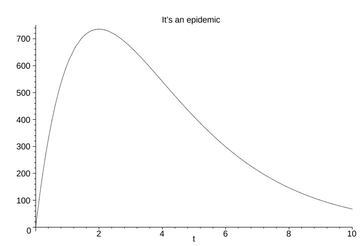
7.6#3 A table of simpsom values for different n via TI-89 and Maple. Looking at the trends it looks like we were within 0.001 for $n = 3$

n	$TI-89$	$Maple$
1	4.2539	4.253895009
2	4.23811	4.238106772
3	4.2368	4.236800683
4	4.23661	4.236613761
5	4.23656	4.236564539
6	4.23655	4.236547143
7	4.23654	4.236539751
8	4.23654	4.236536180

In the table below the approx error is $SIMP(2n) - SIMP(n)$ and the next error for twice n is 1/16 of the approx error. Looking at the error estimates it looks like the error at $n = 2$ is expected to be 0.001 which is too close 0.001 to trust, but by $n = 4$ we are really safe to say we are within 0.001 of the true value.

n	$Maple$	approxerror	nexterrorfortwice$n$
1	4.253895009	$-.015788237$	$-.0009867648125$
2	4.238106772	$-.001493011$	$-.00009331318750$
4	4.236613761	$-.000077581$	$-.000004848812500$
8	4.236536180		

7.7#30 (a) We plot the graph below. (b) r is at its height when $\frac{dr}{dt}$ is zero. $r' = 1000e^{-0.5t} - 500te^{-0.5t} = (1000 - 500t)e^{-0.5t}$. Since e^x is never zero, the max occurs at $t = 2$ days. (c) This is asking for $\int_0^\infty r\,dt$ and since $\int r\,dt = -2000te^{-0.5t} - 4000e^-0.5t$ and $\lim_{x\to\infty} xe^{-x/2} = \lim_{x\to\infty} x/e^{x/2} = \lim_{x\to\infty} 1/(0.5e^{0.5x}) = 0$. The improper integral is 4000 which is the number of people that got sick.



7.8#8 We compare this to $e^y \le e^y + 1$ so $0 \le 1/(1+e^y) \le 1/e^y$ and hence

$$\int_0^\infty \frac{dy}{1+e^y} \le \int_0^\infty e^{-y}dy = lim_{M\to\infty}\int_0^M e^{-y}dy = lim_{M\to\infty} - e^{-y}|_0^M lim_{M\to\infty}(-e^{-M}+1) = 1 < \infty.$$

So the integral converges.