MAC 2312 Calculus 2 **Test 2** 25 Oct 2001

Show **ALL** work for credit; be neat; and use only **ONE** side of each page of paper. Do **NOT** write on this page. Calculators can be used for graphing and calculating only. Give exact answers when possible.

1. A one meter rod has density function $\rho(x) = 30 - 12x kg/m$ where x is the distance from the left end. Find the mass (in kg) and the center of mass (in m) of the rod.

2. Find the radius of convergence for the series below.

$$\sum_{n=0}^{\infty} \frac{2^n x^n}{n!}$$

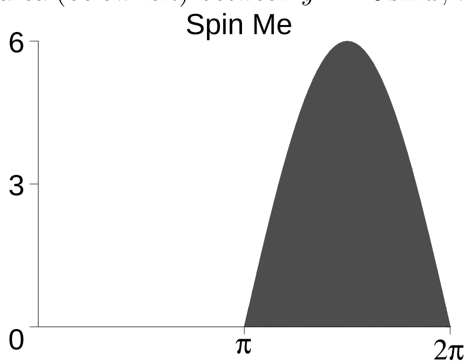
3. Find the radius of convergence for the series below.

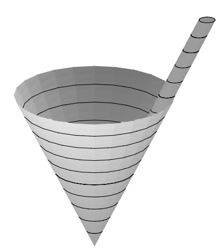
$$\sum_{n=0}^{\infty} \frac{(2n)!x^n}{(n!)^2}$$

4. Suppose that the power series $\sum_{n=0}^{\infty} C_n x^n$ converges for $x = -4$ and diverges when $x = 7$. Which of the following are true, false or not possible to determine? **Give REASONS** for your answers.

a. The power series converges when $x = 10$.
b. The power series converges when $x = 3$.
c. The power series diverges when $x = 1$.
d. The power series diverges when $x = 6$.

5. Write but do **NOT** evaluate integrals which will compute the volume of the figure obtained by rotating the area (below left) between $y = -6\sin x$, $\pi \le x \le 2\pi$ and the x-axis about A. The x-axis. B. The y-axis.





6. Find the work (in N-m (Newton-meters)) done pumping all the water out of a full tank (above right) using a straw of whose top is at 5 meters. The shape of the tank is a cone that has height 3 meters and diameter 3 meters at the top. The density of water, ρ, is $1000 kg/m^3$ and g, acceleration due to gravity, is $9.8 m/s^2$.

7. Write and evaluate the integral which yields the arclength of the one quarter of the unit circle $x^2 + y^2 = 1$ in the first quadrant.

8. For the series $\sum_0^\infty 1/3^n$

A. Find the sum.

B. Show how the ratio test can be used to show the series converges.

C. Show how the integral test can be used to show the series converges.

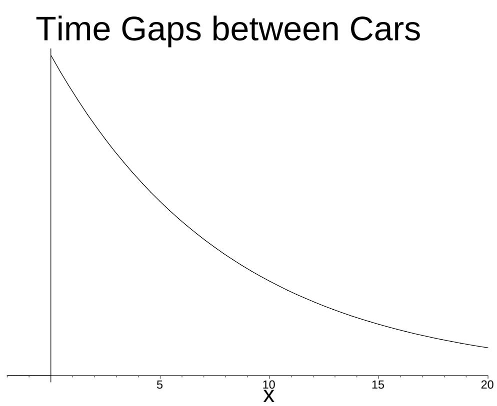
D. Use the series to show $\sum_0^\infty n/(n3^n + \pi)$ converges by the comparison test.

9. Below are three power series that you can assume have radius of converence equal to one. Your job is to check the endpoints x=1 and x=-1 for convergence. Write down the resulting (non-power) series at each endpoint, write down whether each endpoint series converges or diverges and finally give the interval of convergence of the power series. To make the problem more interesting, there are number of other bits of information about each series, but nothing more is required of you.

$$A.\ \sum_1^\infty \frac{(-1)^{n+1}x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} + \cdots = \int_0^x 1 - t + t^2 + \cdots \ dt = \int_0^x \frac{1}{1+t}\, dt = \ln(1+x)$$

$$B.\ \sum_0^\infty nx^n = x + 2x^2 + 3x^3 + \cdots = x\frac{d}{dx}(1 + x + x^2 + x^3 + \cdots) = x\frac{d}{dx}\frac{1}{1-x} = \frac{x}{(1-x)^2}$$

$$C.\ \sum_1^\infty \frac{x^n}{n^2} = \int_0^x 1 + t/2 + t^2/3 + \cdots \ dt = \int_0^x \frac{t + t^2/2 + t^3/3 + \cdots}{t}\, dt = \int_0^x \frac{\ln(1-t)}{t}\, dt = \text{polylog}(2, x)$$



10. An experiment was done observing the time gaps between cars on Tennessee Street. The data shows that the probability density function of these time gaps was given approximately by $p(x) = \alpha e^{-0.122x}$, where $x \geq 0$ is time in seconds (see above).

a. Find α.

b. Find and sketch P, the cumulative distribution function.

c. Find the median time gap.

d. Find the mean time gap (use TI-89 to integrate).