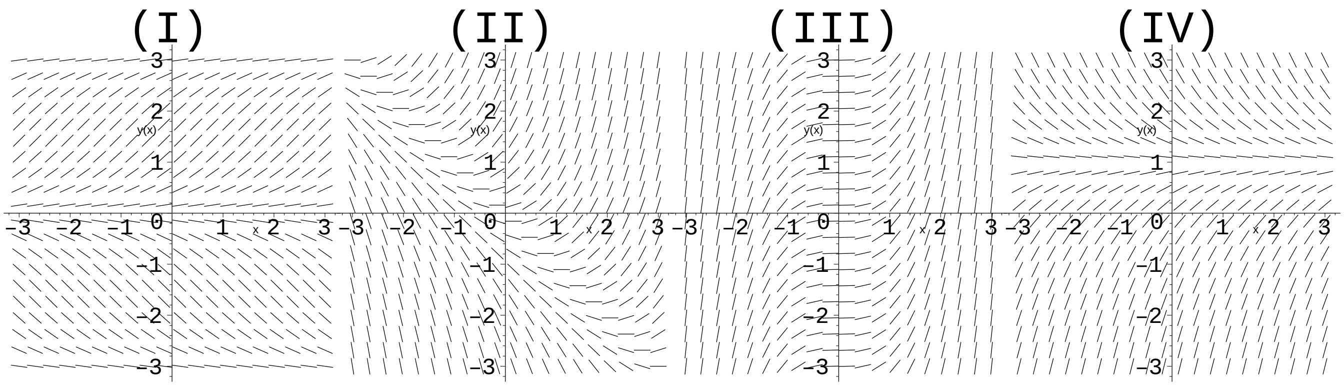
MAC 2312 Calculus 2 **Test 3** 29 Nov 2001

Show **ALL** work for credit; be neat; and use only **ONE** side of each page of paper. Do **NOT** write on this page. Calculators can be used for graphing and calculating only. Give exact answers when possible.

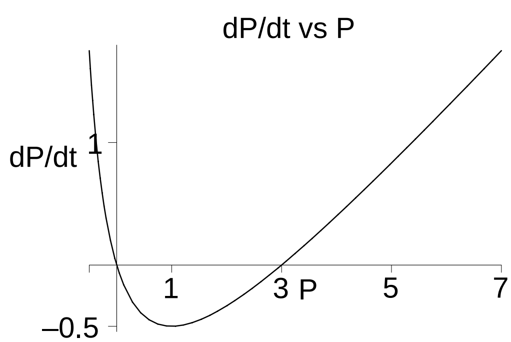
1. Match the slope fields below with the differential equations $y' = x^2, y' = \sin y, y' = 1 - y$ and $y' = x + y$.

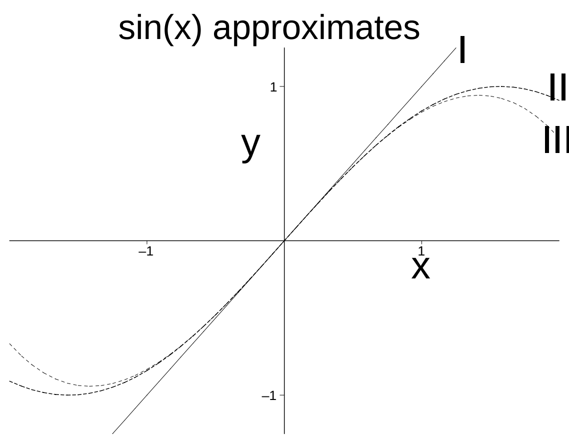


2. Find all values of r so that $y = x^r$ is a solution to $x^2y'' + 2xy' - 6y = 0$.

3. Find the solution of $y' = -y/x,\ y(1) = -2$. Sketch the graph of your solution.

4. For the differential equation $\frac{dP}{dt} = f(P)$, the graph of $f(P)$ or $\frac{dP}{dt}$ versus P is given below (left).
   a. Sketch a graph of the slope field for this differential equation.
   b. Find both equilibrium solutions, and label them as stable or unstable.
   c. On your slope field, find and sketch a solution with an inflection point, and label your inflection point with its coordinates.





5. Approximating $\sin x$ by x and $x - x^3/3!$.
   a. The graph to the above (right) graphs these three functions, identify which is which. [Hint: What is the next term in the Taylor series?]
   b. Use $\sin x \approx x - x^3/3!$ to show
$$\lim_{x\to 0} \frac{\sin x}{x} = 1$$
.
   c. Use $\sin x \approx x - x^3/3!$ to estimate (to five significant digits) the relative error $= (\sin x - x)/\sin x$ of using x to approximate $\sin x$ for the angle x in radiams that corresponds to $15°$. [So you are using the better approximation to gauge the error in the simpler approximation.]

6. Consider the IVP

$$y' = 1 + y^2, y(0) = 1$$

(a) Use Euler's method by hand with two steps to estimate $y(1)$.
(b) Sketch the slope field for this differential equation in the first quadrant, and use it to decide if your estimate is an over- or underestimate.
(c) Use Euler's method via your calculator to estimate $y(1)$ with ten steps.

7 & 8. These problems are about the Taylor series for the functions f and g given below.

$$f(x) = \sum_{n=1}^{\infty} nx^n = x + 2x^2 + 3x^3 + 4x^4 + \dots$$

$$g(x) = \sum_{n=1}^{\infty} 2^{n-1}x^n = x + 2x^2 + 4x^3 + 8x^4 + \dots$$

a. Is f or g larger for small positive x and why?
b. Find $g'''(0)$.
c. Using substitution, find the Taylor series for $f(2u)$.
d. Find the Taylor series for $g'(x)$.
e. Show the multiplication needed to get the first 4 non-zero terms of the Taylor series for fg.

9. A Calculus class at a party school brings an ice cream cake to a 7:30 final. The cake is frozen ($40°F$) too hard to eat right away. Besides the class is eager to take the final. Two hours later the cake is at $50°F$ and is eaten immediately. The classroom is at a constant $70°F$.
a. Assuming the temperature, T, of the cake obeys Newton's Law of Cooling, write a differential equation for T.
b. Solve the differential equation to estimate the time the cake was taken out of a $30°F$ freezer.

10. Santa is making a list. He is adding names at the rate of 1 million names a day and 99 % of the new names are "nice". At the same time 1 million names a day are randomly selected to fall off the list. (The list always has the same number of names.) The list starts with 1 billion people, 95 % of whom are nice. Derive a differential equation and initial conditions for N the number of "nice" people on the list. Do **NOT** solve your IVP. [Hint: It is like salt in water.] Be sure to check your list twice.