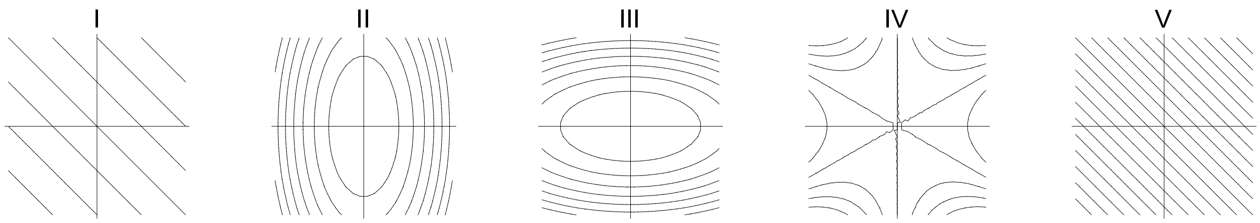
MAC 2313 Calculus 3 **Test 1** 17 Sep 2002

Show **ALL** work for credit; be neat; and use only **ONE** side of each page of paper. Do **NOT** write on this page. Calculators can be used for graphing and calculating only. Give exact answers when possible.

1. Find the equation of the plane parallel to the plane $2x - y - 5z = 5$ and passing through the point (-1,2,-3).

2. Find the equation of the plane through the points (1, -1, 0), (1, 1, 1) and (0, 1, 2).

3. The following are contour plots of $z = 4x^2 + y^2$, $z = x^2 + 4y^2$, $z = x^3 - 3xy^2$, $z = x + y$, and $z = 3x + 3y$ over the range $x = -3..3$ and $y = -3..3$. Match the plot to the function.



4. Find the point of intersection of the two lines below or show they do not intersect.

$$\ell_1 : x = 5 + 2t \qquad y = 5 + t \qquad z = -2 + 3t \qquad \text{and} \qquad \ell_2 : x = 4 - 3t \qquad y = 1 - 5t \qquad z = 2 + t$$

5. Find the equation of the line through the origin perpendicular to the plane $x - 2y + 2z = 18$. Find P, the point of intersection. Find the distance from the origin to P.

6. An ant is standing at the origin O of a flat plain and looks to an apple at point A in a tree. The ant crawls to the apple avoiding an obstacle. The ant starts along a line making a angle of $\pi/6$ with the x-axis (slightly east of northeast) for 10 meters. The ant then turns north (parallel to the y-axis) and travels another 10 meters. Finally the ant climbs 2 meters straight up the tree and arrives at the point A. Find the vector $\overrightarrow{OA}$ its length and a unit vector pointing in the same direction.

7. Check your calculator and make sure it is in radian mode. Let $\mathbf{r}(t) = \langle \cos(t), \sin(6t) \rangle$
(a) Find the velocity and acceleration of $\mathbf{r}(t)$.
(b) Plot the curve for $0 \le t \le 2\pi$ [Hint: use the TI-89.]
(c) Write an integral which will give the arclength of the curve and find a numerical approximation for the integral. [Calculator]

8. Write $\mathbf{a} = \langle 3, 2, -5 \rangle$ as the sum of two vectors, one parallel, and one perpenducular to $\mathbf{d} = \langle 2, -2, 1 \rangle$. Use your answer to find the distance from the point (3, 2, -5) [the endpoint of **a**] to the line $\langle x, y, z \rangle = \langle 2t, -2t, t \rangle$. [the line in the direction of **d** and through the origin.]

9. Let $\mathbf{X} = \langle x, y, z \rangle$, $\mathbf{A} = \langle -1, -3, 2 \rangle$, $\mathbf{B} = \langle 11, 1, -1 \rangle$, $\mathbf{C} = \langle 1, 1, 1 \rangle$, and $\mathbf{D} = \langle 3, -1, 2 \rangle$. Find the vector projection of $\mathbf{B} - \mathbf{A}$ onto $\mathbf{C} \times \mathbf{D}$ and call it $\mathbf{E}$. In general, so long as $\mathbf{C}$ and $\mathbf{D}$ are not parallel, the lines $\mathbf{X} = \mathbf{A} + \mathbf{E} + t\mathbf{C}$ and $\mathbf{X} = \mathbf{B} + t\mathbf{D}$ now intersect. Using the numbers above where do they intersect?

10. Plot the contour lines for the equation $x^2 + (y - z)^2 = 4$ for the z values $z = 0$, 1, 2, and 3. Label your contours with the z values. [Since the equation is not a function $z = f(x, y)$, it is ok for the contours to intersect.] On a separate graph give a 3D sketch of the surface and describe the graph in words.