MAC 2311 Calculus 1 **Test 4** 26 Oct 2004

Directions: Show **ALL** work for credit; Give **EXACT** answers when possible; Start each problem on a **SEPARATE** page; Use only **ONE** side of each page; Be neat; Leave margins on the left and top for the **STAPLE**; Calculators can be used for graphing and calculating only; Nothing written on this page will be graded;

1. For $f(x) = x^3 + x - 1$, $a = 0$ and $b = 2$, find all c that satisfies both conclusions of the Mean Value Theorem, one of which is
$$\frac{f(b) - f(a)}{b - a} = f'(c)$$

2. Find the limits.
$$(a)\ \lim_{x\to 0} \frac{5^x - 3^x}{x} \qquad\qquad (b)\ \lim_{x\to 0} \frac{x - \sin x}{x^3}$$

3. Find the absolute maximum and absolute minimum **VALUES** of $f(x) = x/(x^2 + 1)$ on $[-3, 2]$.

4. True or False and a brief reason why or why not.
   (a) $\lim_{x\to\infty} x^{1000}/e^x = 0$
   (b) If $f'(x) < 0$ for $x < c$ and $f'(x) > 0$ for $x > c$, then $(c, f(c))$ is a local maximum for $y = f(x)$
   (c) If $f'(c) = 0$ and $f''(c) > 0$, then $(c, f(c))$ is a local maximum for $y = f(x)$.
   (d) There exists a function such that $f(x) < 0$, $f'(x) > 0$ and $f''(x) < 0$ for all x.
   (e) There exists a function such that $f(x) > 0$, $f'(x) > 0$ and $f''(x) > 0$ for all x.
   (f) If $f'(c) = f''(c) = 0$, then $x = c$ is neither a local minimum nor a local maximum.
   (g) If $f'''(x) > 0$ for all x and $f''(c) = 0$, then $(c, f(c))$ is a point of inflection for $y = f(x)$.
   (h) $\lim_{x\to\pi^-} \sin x/(1 - \cos x) = \lim_{x\to\pi^-} \cos x/ \sin x = -\infty$
   (i) If $f'(x)$ exist for all x, then between any two zero's of $f(x)$ there is a zero of $f'(x)$.
   (j) If $f(c) = f'(c) = 0$ and $f''(x) > 0$ for all x then $f(x) > 0$ for $x > c$.

5. Graph the function $y = f(x)$ given the data below, label all local minimums, local maximums and points of inflection.
$f(0) = 0$, $\lim_{x\to-\infty} f(x) = 1$, and $\lim_{x\to\infty} f(x) = 3$
$f'(x) > 0$ for $(-\infty, -2)$, $(0, 2)$ and $(2, \infty)$ and $f'(x) < 0$ for $(-2, 0)$
$f''(x) > 0$ for $(-\infty, -3)$, $(-1, 1)$ and $(2, 3)$ and $f''(x) < 0$ for $(-3, -1)$, $(1, 2)$ and $(3, \infty)$.