Problems on Generalized Functions (part of HW#7)

1. Let $f_n(x) = n\chi_{[1/n,2/n]}$ and T_n the distribution associated with f_n.

   (a) Show $T_n \to \delta$.
   (b) Find the derivatives of T_n
   (c) Show $T_n' \to \delta'$.

2. Show

   (a) $\delta(ax) = \delta(x)/|a|$
   (b) $f(x)\delta(x) = f(0)\delta(x)$
   (c) $f(x)\delta'(x) = -f'(0)\delta(x)$

3. Give an example of two sequences of functions (a_n) and (b_n) with associated distributions (α_n) and (β_n) so that both sequences converge to the distribution δ, but so that $\langle T, \phi \rangle = \lim_n \int_{-\infty}^{\infty} a_n(t)b_n(t)\phi(t)\,dt$ converges but to something different then $\phi(0)^2$.