# Fun Exercises 3

For maa4402

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These are just more problems like hw11.1

1. Without evaluating the integral, show that

$$\left| \int_C \frac{dz}{z^2 - 1} \right| \leq \frac{\pi}{3}$$

when $C$ is the arc of the circle $|z| = 2$ from $z = 2$ to $z = 2i$ that lies in the first quadrant.

2. Let $C$ be the line segment from $z = i$ to $z = 1$. By observing that the midpoint is the closest point to the orgin, show

$$\left| \int_C \frac{dz}{z^2} \right| \leq 4\sqrt{2}$$

3. Let $C$ be the circle $|z| = R$ $(R > 1)$ in a counterclockwise direction from $-R$ back to $-R$ $(-\pi < \theta < \pi)$. Show that

$$\left| \int_C \frac{\operatorname{Log} z}{z^2} dz \right| \leq 2\pi \left( \frac{\pi + \ln R}{R} \right)$$

and then show that the integral tends to zero as $R$ tend to infinity. (Hint: L'Hospitial.)

4. Find an upper bound for

$$\left| \int_C \frac{dz}{(z^2 + 1)} \right|$$

where $C$ is the upper half-circle $|z| = R$ with radius $R > 1$.

*answer* $\pi R/(R^2 - 1)$

5. $C$ is the square with corners $R$, $Ri$, $-R$ and $-Ri$ oriented counterclockwise. Find an upper bound to

$$\left| \int_C f(z) dz \right|$$

when $f(z)$ has the given value.

   (a) $f(z) = z^4 + 4z^2 + 4$
   (b) $f(z) = (z^4 + 4z^2 + 4)^{-1}$
   (c) $f(z) = \overline{z}^4 + 4\overline{z}^2 + 4$
   (d) $f(z) = (\overline{z}^4 + 4\overline{z}^2 + 4)^{-1}$