MAC 3313 Calculus 3 **Test 3** 26 Nov 1996

Show **ALL** work for credit; be neat; and use only **ONE** side of each page of paper.

1. Find the curl and div of $\mathbf{F} = \langle xe^y, -ze^{-y}, y\ln z\rangle$.

2. Find f so that $\mathbf{F} = \nabla f$ and use it to find the line integral $\int_C \mathbf{F}\cdot d\mathbf{r}$. Here $\mathbf{F} = \langle y, x+z, y\rangle$. and C is a curve from $(2,1,4)$ to $(8,3,-1)$.

3. Find the equation of the tangent plane to $\mathbf{r}(u,v) = \langle uv, ue^v, ve^u\rangle$ when $u_0 = 1, v_0 = 0$. Compute x_0, y_0, z_0.

4. Write down and simplify but do **NOT** evaluate the double integral with polar co-ordinates from using Green's Theorem to change $\int_C xy dx + 2x^2 dy$ to a double integral. C consists of the line segment from (0,1) to (0,0), then the line segment from (0,0) to (1,0) and then upper right quarter of the unit circle $x^2+y^2=1$.

5. Write down and simplify but do **NOT** evaluate the integral (in terms of t) to find the mass of a thin wire bent into the shape of the semi-circle $x^2+y^2=4, x\geq 0$ if the density is given by $\rho(x,y)$.

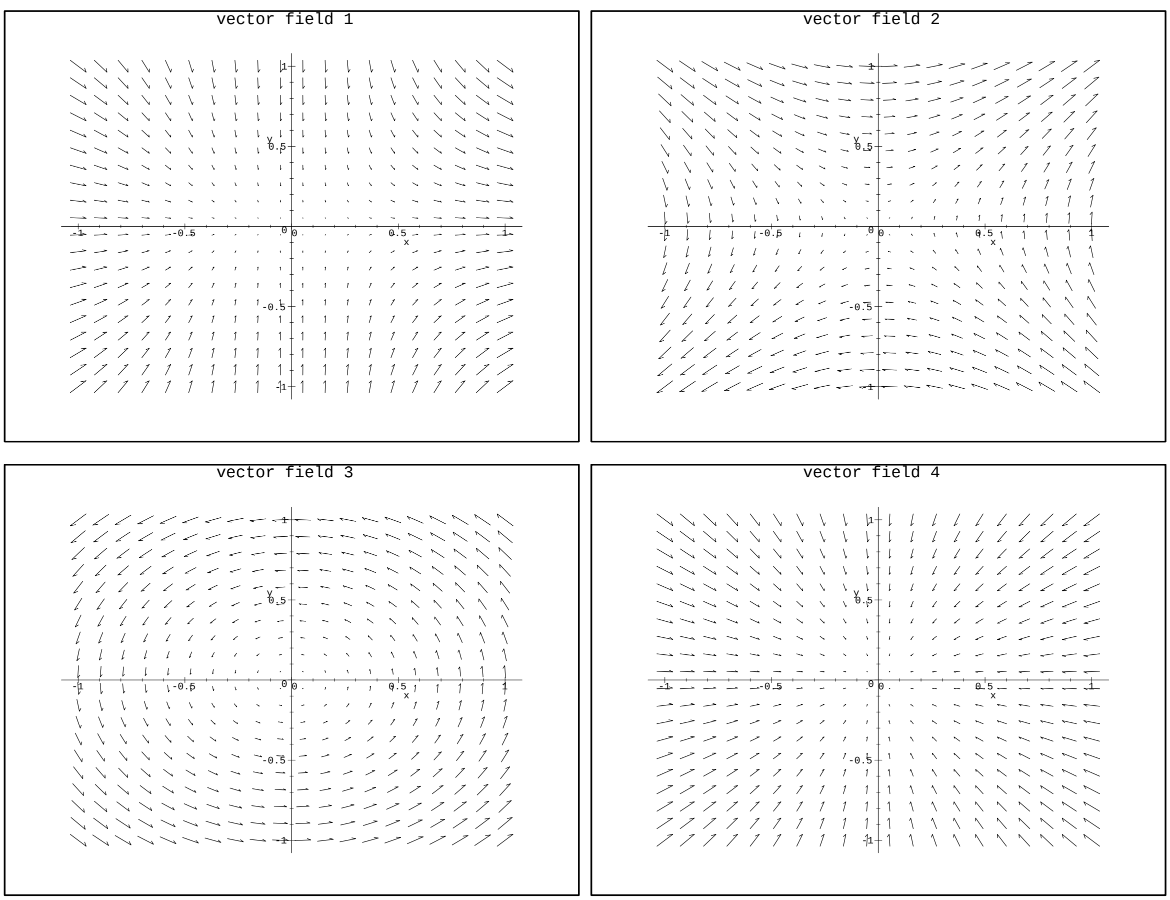
6. Write down and simplify but do **NOT** evaluate the surface integral $\int\int_S \mathbf{F}\cdot d\mathbf{S}$. Where S is given by $\mathbf{r}(u,v) = \langle u\cos v, u\sin v, v\rangle$, $0\leq u\leq 1, 0\leq v\leq\pi$ and $\mathbf{F} = \left\langle y, x, z^2\right\rangle$.

7. Use spherical co-ordinates and density $f(x,y,z) = z$ to find the mass of the part of the ball $x^2+y^2+z^2\leq 1$ in the first octant.

8. Re-write the integral $\int_0^1\int_y^1\int_0^y f(x,y,z)dzdxdy$ in the orders $dzdydx$ and $dydxdz$.

9. Evaluate $\int\int_R e^{x+y}dA$ where R is the "diamond" region $|x|+|y|\leq 1$ using the transformation $x = (u+v)/2, y = (u-v)/2$. Explicitly draw the regions R and S. Clearly label the Jacobian of the transformation.

10. Match the vector fields $\langle y,x\rangle$, $\langle -y,x\rangle$, $\left\langle x^2,-y\right\rangle$, and $\langle -x,-y\rangle$ with the Maple fieldplots below.



Maple fieldplots