MAC 2311 Calculus 1 **Test 2** 19 Oct 1999

Show **ALL** work for credit; be neat; and use only **ONE** side of each page of paper. Do **NOT** write on this page. Calculators can be used for graphing and calculating only. Give exact answers when possible.

1. Find $\int_1^2 x^x dx$ using your calculator. (State the numerical method you used and the model of your calculator.)

2. Find an equation for the tangent line to $f(x) = 1/x$ at $x = 2$. Plot $f(x)$ and this tangent line.

3. You jump out of an airplane. Before your parachute opens you fall faster and faster, but your acceleration decreases as you fall because of air resistance. The table below gives your acceleration, a (in m/sec^2), after t seconds.

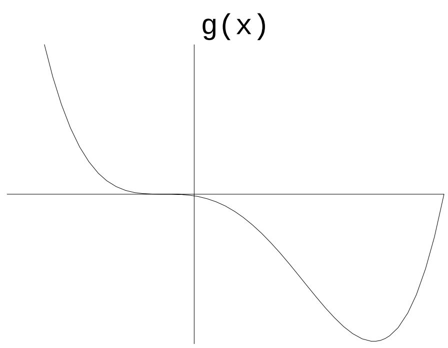
t(seconds)	0	1	2	3	4	5
$a(t)$(m/sec^2)	9.81	8.03	6.53	5.38	4.41	3.61

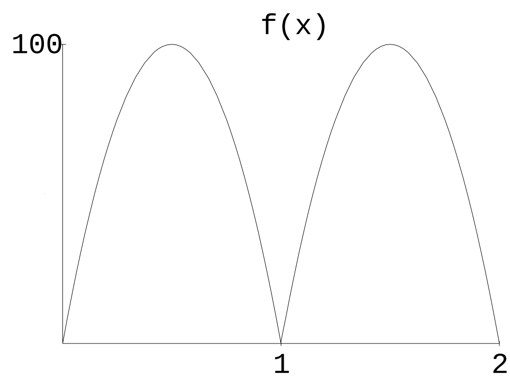
(a) Give upper and lower estimates of your speed at $t = 5$.

(b) Get a new estimate by taking the average of your upper and lower estimates. What does the concavity of the graph of acceleration tell you about your new estimate?

4. The temperature, T, in degrees Fahrenheit, of a cold yam places in a hot oven is given by $T = f(t)$, where t is the time in minutes since the yam was put in the oven. What is the sign of $f'(t)$ and why? What are the units of $f'(20)$? What is the practical meaning of the statement $f'(20) = 2$?

5. Sketch the graph of the derivative to the function $g(x)$ in the graph below. (You might want to trace $g(x)$ onto your answer sheet.)





6. The graph of the continuous function $f(x)$ is given above. Rank the following integrals in ascending numerical order. Explain your reasons.

(i) $\int_0^2 f(x)dx$ (ii) $\int_0^1 f(x)dx$ (iii) $\int_0^2 (f(x))^{1/2}dx$ (iv) $\int_0^2 (f(x))^2 dx$.

7. There is a function called the error function, $y = erf(x)$. Suppose your calculator has a button for $erf(x)$ that gives the following values:

$$erf(0) = 0 \qquad erf(1) = 0.84270079 \qquad erf(0.1) = 0.11246292 \qquad erf(0.01) = 0.01128342$$

(a) Use all this information to determine your best estimate for $erf'(0)$. (Give only those digits of which you feel reasonably certain.)

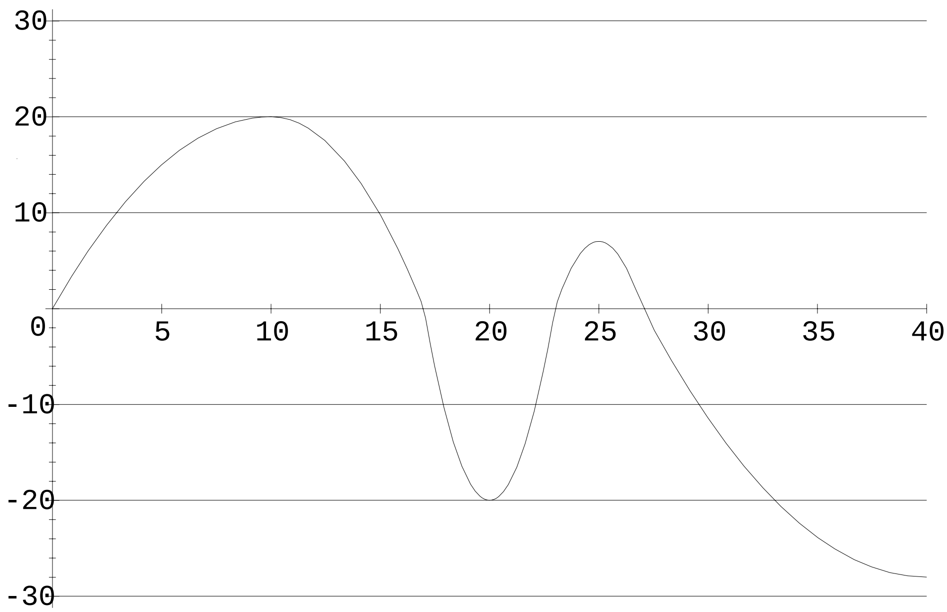
(a) Suppose you find that $erf(0.001) = 0.00112838$. How does this extra information change your answer to part (a)?

There is more test on the other side.

Welcome to test two side two.

8. Find the derivative of $f(x) = x^3$ algebraically (i.e. using the limit definition).

9. Graph an increasing concave up function $y = F(x)$ from $x = a$ to $x = b$ and suppose $F'(x) = f(x)$. Mark the following quantities on your graph.
(a) A slope representing $f(a)$.
(b) A length representing $\int_a^b f(x)dx$.
(c) A slope representing $\frac{1}{b-a}\int_a^b f(x)dx$.



10. A mouse moves back and forth in a tunnel attracted to bits of cheddar cheese alternately introduced to and removed from the ends (right and left) of the tunnel. The graph of the mouse's velocity, v in cm/sec, is given in the graph above vs the time, t in seconds, with the positive velocity corresponding to the motion toward the right end. Assuming the mouse starts ($t = 0$) at the center of the tunnel, use the graph to estimate the time(s) at which
(a) The mouses changes direction.
(b) The mouse is moving most rapidly to the right; to the left.
(c) The mouse is farthest to the right of center; farthest to the left.
(d) The mouse's speed (i.e. the magnitude of its velocity) is decreasing.
(e) The mouse is at the center of the tunnel.