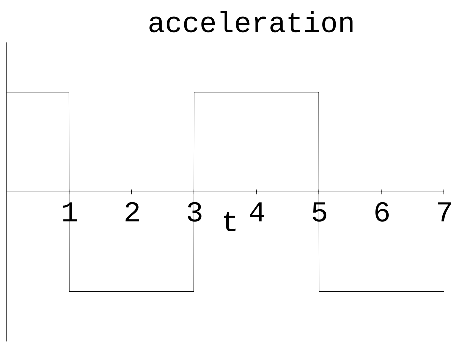
MAC 2311 Calculus 1 **Test 3** 2 Dec 1999

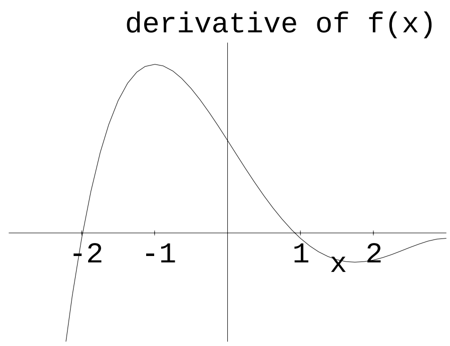
Show **ALL** work for credit; be neat; and use only **ONE** side of each page of paper. Do **NOT** write on this page. Calculators can be used for graphing and calculating only. Give exact answers when possible.

1. Let $f(x) = \sin(x) + 2^x + \sinh(x) + \sqrt{x} + \pi$
   a. Find $f'(x)$.
   b. Find $\int f(x)dx$.

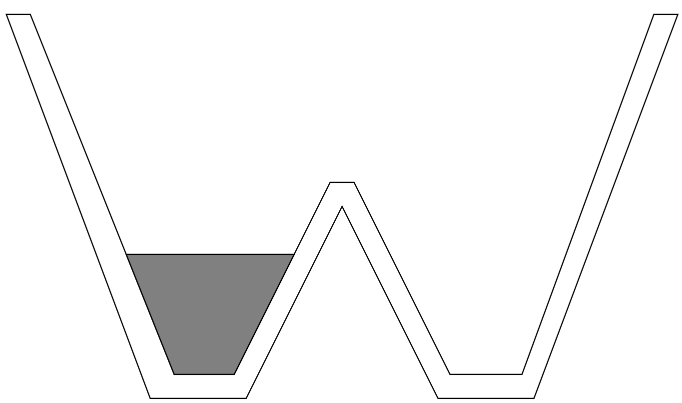
2. Given $r(2) = 4, s(2) = 1, s(4) = 2, r'(2) = -1, s'(2) = 3, s'(4) = 3$. Compute the following derivatives, or state what additional information you would need to be able to compute the derivative.
   a. $H'(2)$ if $H(x) = r(x) + 2s(x)$
   b. $H'(2)$ if $H(x) = r(x) \cdot s(x)$
   c. $H'(2)$ if $H(x) = \sqrt{r(x)}$
   d. $H'(2)$ if $H(x) = r(s(x))$
   e. $H'(2)$ if $H(x) = s(r(x))$

3. The acceleration, a, of a particle as a function of time t is shown in the graph below (left). Sketch graphs of the velocity and position against time. Assume the particle starts at rest at the origin.





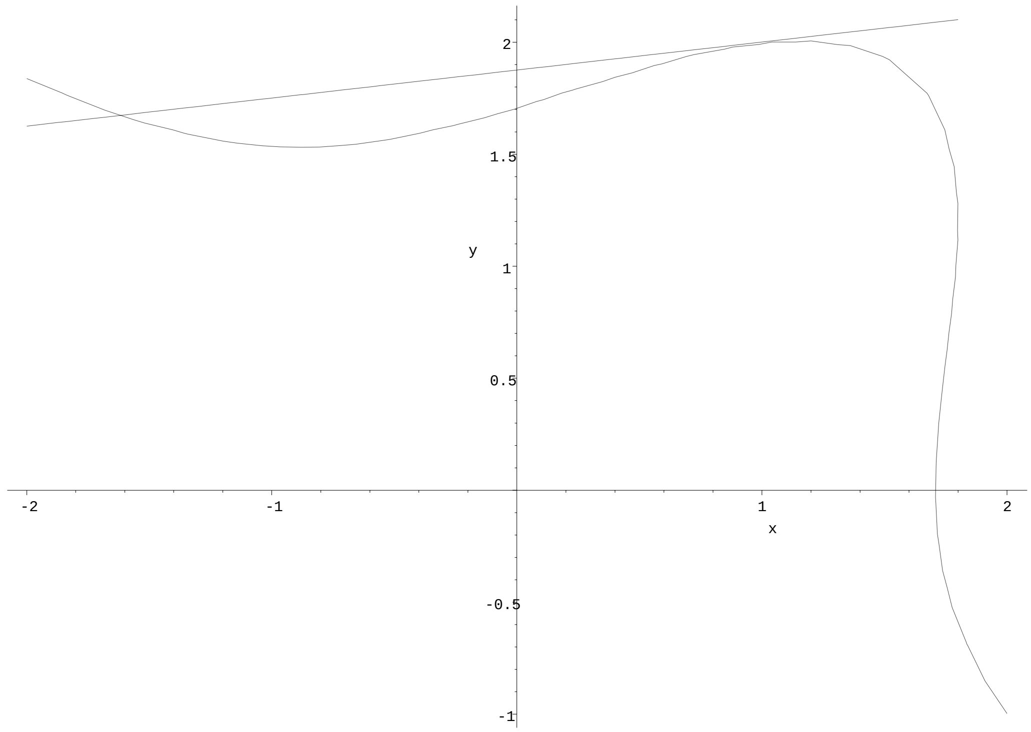
4. The graph above (right) plots the derivative of the function $f(x)$. Sketch a possible graph for $f(x)$. Mark the points, $x = -2, -1, 1, 2$ on your graph and label local maxima, local minima and points of inflection.



5. Water flows at a constant rate into the left side of the W-shaped container shown above. Sketch a graph of the the height H, of the water in the left side of the container as a function of time, t. Suppose the container starts out empty. [Make sure your function has the proper concavity. Yes, the water does start filling the right side when the height is high enough.]

There is more test on the other side.

Welcome to test three side two.

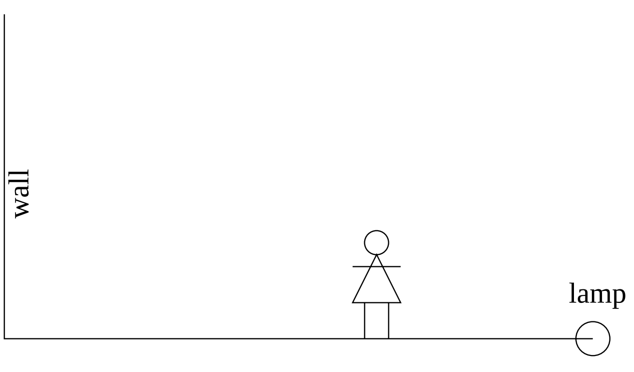


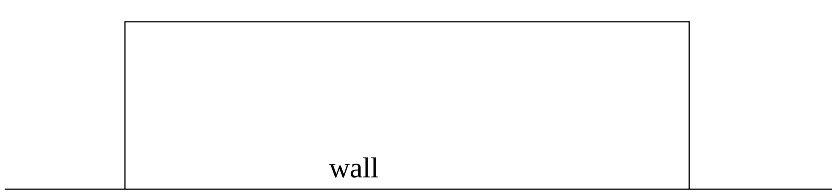
6. Consider the equation $x^3 + y^3 - xy^2 = 5$ (see graph above).
   a. Find dy/dx by implicit differentiation.
   b. Find the equation of the tangent line to the curve when $x = 1$ and $y = 2$, (see graph above).

7. Find $\lim_{x \to 0^+} x \ln x$ using L'Hopital rule. [Hint: $x \ln x = \frac{\ln x}{1/x}$.]

8. Find the (global) minimum and maximum values of the function $f(x) = x + \sin x$ for $0 \leq x \leq 2\pi$.

9. A floor lamp is 30 feet away from a high wall and directly between the lamp and the wall is a 6 foot women 20 feet from the wall (see below left). How fast is her shadow changing if the lamp is moving 2 feet per second away from the wall?





10. If you have 100 feet of fencing and want to enclose a rectanglar area up against a long, straight wall, (see above right,) what is the largest area you can enclose?