MAC 2313 Calculus 3 **Test 3** 13 Apr 2000

Show **ALL** work for credit; be neat; and use only **ONE** side of each page of paper. Do **NOT** write on this page. Calculators can be used for graphing and calculating only. Give exact answers when possible.

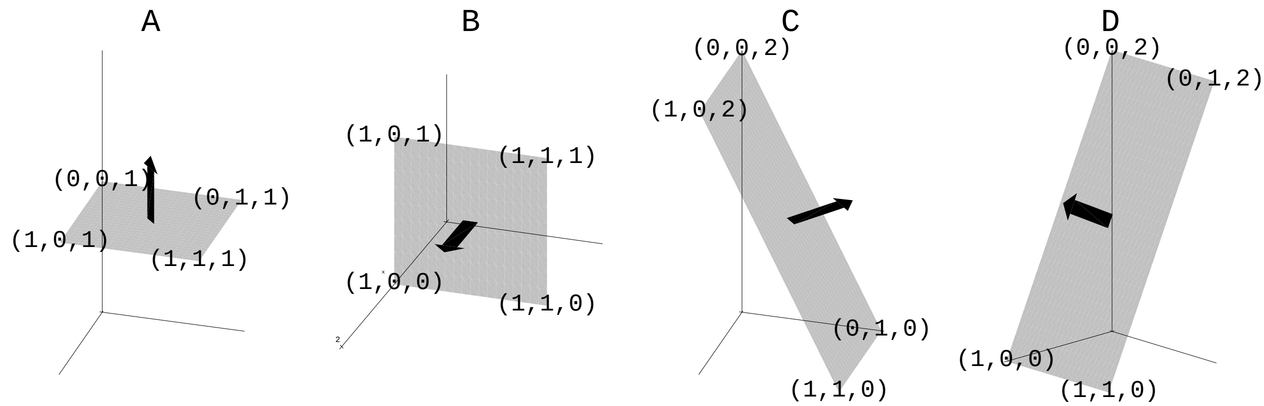
1. For the vector field $\mathbf{F} = \left\langle 2xy^3 + y, 3x^2y^2 + x \right\rangle$ find $\nabla \cdot \mathbf{F}$ (that is, div $\mathbf{F}$) and either find a scalar field f so that $\mathbf{F} = \nabla f$ (that is, grad f) or show no such f exists.

2. For $\mathbf{r}(t) = \left\langle t, t^2, t^3 \right\rangle$.
 A. Find the velocity and acceleration of $\mathbf{r}(t)$.
 B. Use numerical integration on your calculator to compute the length of this curve from t=0 to t=2.

3. Evaluate the integral below by changing to cylindrical coordinates.

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{x^2+y^2}} (z + \sqrt{x^2+y^2}) \quad dzdydx$$

4. Compute the flux of the vector field $\mathbf{F} = \langle 2, 3, 5 \rangle$ through each of the rectangular regions in (A)-(D), assuming each is oriented as shown.



5. For the integral below, sketch the region of integration and then evaluate the integral by reversing the order of integration.

$$\int_0^3 \int_{y^2}^9 y \sin(x^2) \quad dxdy$$

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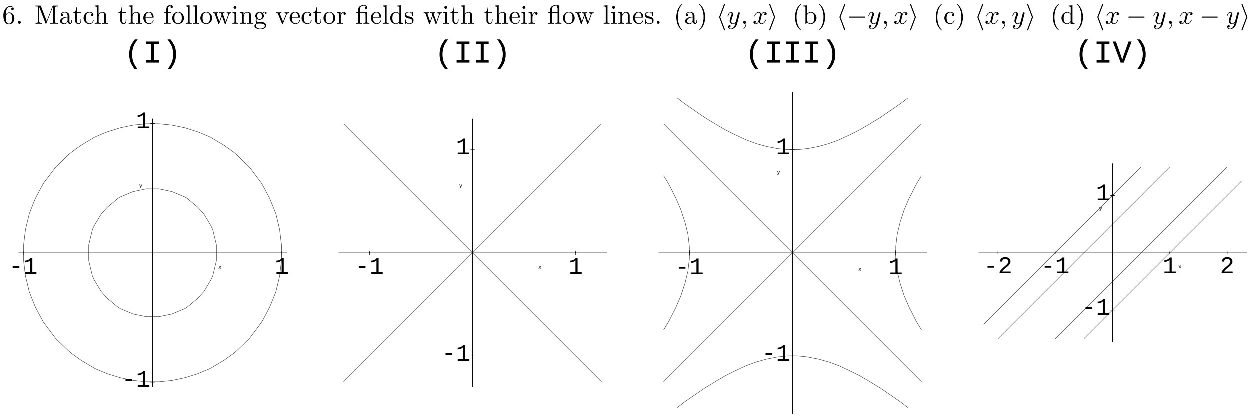
6. Match the following vector fields with their flow lines. (a) $\langle y, x \rangle$ (b) $\langle -y, x \rangle$ (c) $\langle x, y \rangle$ (d) $\langle x - y, x - y \rangle$



There is more test on the otherside

7. Use Green's Theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ if $\mathbf{F} = \langle x^2 - y, y^2 + x^2 \rangle$ and C is the counterclockwise boundry of the rectangle $0 \leq x \leq 2, 0 \leq y \leq 3$.

8. Use the Divergence Theorem to evaluate the flux integral $\int\int_S \langle x^2, y - 2xy, 10z \rangle \cdot d\mathbf{A}$, where S is the sphere of radius 5 centered at the origin, oriented outwards.

9. (Set up and simplify the integral, but do **NOT** evaluate the integral.) Find the flux when $\mathbf{F} = \langle 3x, y, z \rangle$ and S is the surface given by $z = -2x - 4y + 1$ above the triangle in the xy-plane with vertices $(0,0), (0,2)$ and $(1,0)$.

10. Consider the parametric equations for $0 \leq t \leq \pi$.
(I) $\langle \cos(2t), \sin(2t) \rangle$ (II) $\langle 2\cos(t), 2\sin(t) \rangle$ (III) $\langle \cos(t/2), \sin(t/2) \rangle$ and (IV) $\langle 2\cos(t), -2\sin(t) \rangle$.
A. Match the equations above with four of the curves A, B, C, D, E and F in graph below.
B. Give parametric equations for the curves which have not been matched, again assuming $0 \leq t \leq \pi$.