MAC 2312 Calculus 2 **Test 2** 8 Mar 2001

Show **ALL** work for credit; be neat; and use only **ONE** side of each page of paper. Do **NOT** write on this page. Calculators can be used for graphing and calculating only. Give exact answers when possible.

1. New series from old$_1$: Write down the Taylor series for $\cos x$ and use it find the Taylor series for $\cos\sqrt{|x|}$. (Make sure your last answer has no square roots or absolute values.) For small positive x, which is bigger $\cos x$ or $\cos\sqrt{x}$?

2. A two meter rod has density function $\rho(x) = x^2 kg/m$ where x is the distance from the left end. Find the mass (in kg) and the center of mass (in m) of the rod.

3. Find both $h'''(0)$. and the radius of convergence of the power series below

$$h(x) = \sum_{n=0}^{\infty} \frac{n^2 x^n}{3^n} = \frac{x}{3} + \frac{4x^2}{9} + \frac{9x^3}{27} + \frac{16x^4}{81} + \dots$$

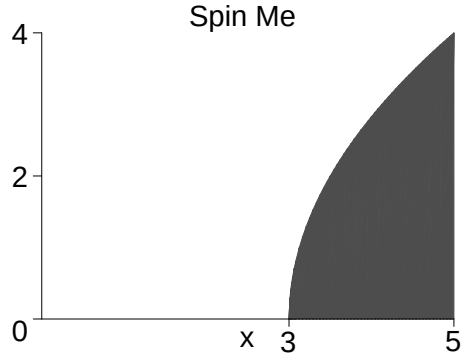
4. Write the integral and use your calculator's numeric integration to evaluate the integral which yields the arclength of one period of the $\sin(x)$ function.

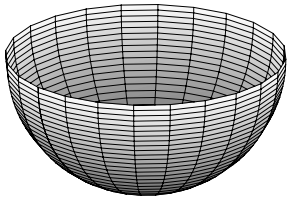
5. New series from old$_2$: You are given the Taylor series for the functions f and g below.

$$f(x) = \sum_{n=1}^{\infty} nx^n = x + 2x^2 + 3x^3 + 4x^4 + \dots$$
$$g(x) = \sum_{n=1}^{\infty} n^2 x^n = x + 4x^2 + 9x^3 + 16x^4 + \dots$$

a. Show the long division needed to get the first 4 non-zero terms of f/g.
b. Find the Taylor series for $\int_0^x f(t)/t \quad dt$.

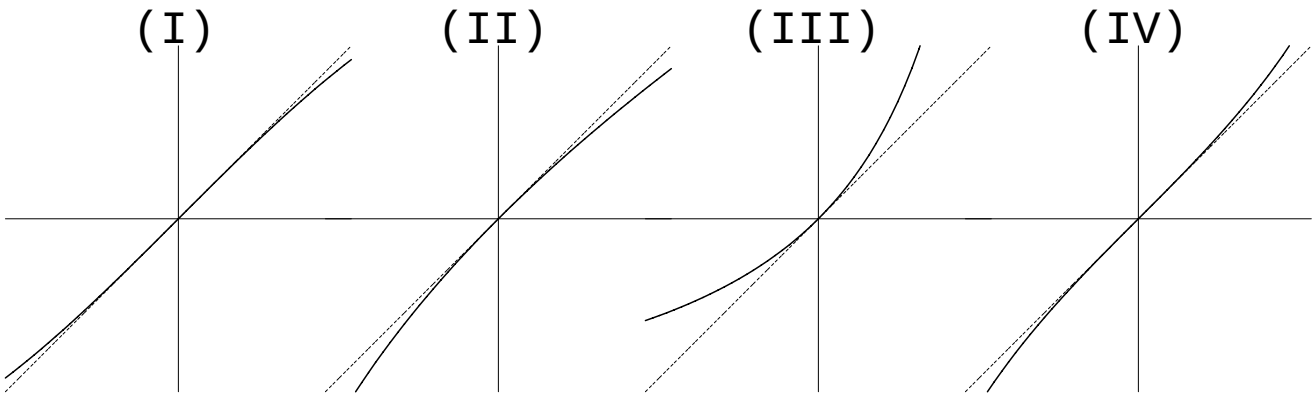
6. Write but do **NOT** evaluate integrals which will compute the volume of the figure obtained by rotating the area (below left) between $y = \sqrt{8(x-3)}$, $3 \le x \le 5$ and the x-axis about A. The x-axis. B. The y-axis.





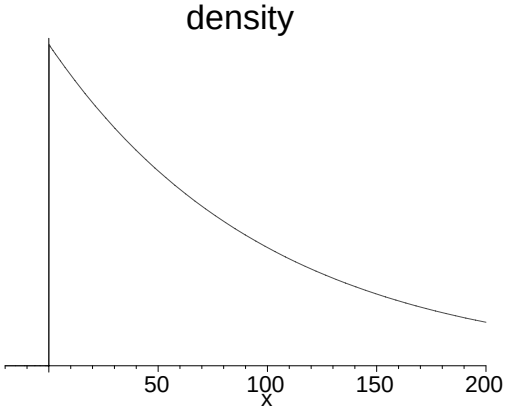
7. Find the work (in N-m (Newton-meters)) done pumping all the water out of a full fish tank (above right) that has the shape of a hemisphere of radius 1 meter. The density of water, ρ, is $1000 kg/m^3$ and g, acceleration due to gravity, is $9.8 m/s^2$.

8. Find the first two non-zero terms of the Taylor series for the functions $\sin x$, $\tan x$, $x/(1-x)$ and $2\sqrt{1+x}-2$ and then match the functions to the graphs below (the thin dotted line is $y=x$).



9. Find exact values for the sum of the following series.

$$A = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots$$
$$B = 1 + 0.1 + \frac{(0.1)^2}{2!} + \frac{(0.1)^3}{3!} + \dots$$
$$C = \frac{42}{100} + \frac{42}{10000} + \frac{42}{1000000} + \frac{42}{100000000} + \dots$$
$$D = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots$$
$$E = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$$



10. The failure rate of a electronic device often has a decaying exponential probability density $\rho(x) = \lambda e^{-\lambda x}$ for $x \geq 0$ and $\rho(x) = 0$ for $x < 0$ (see above). For this problem, let $\lambda = 1/100$ and let x be time in seconds.

a. Find the percent of failures in the first 3 seconds.
b. Find the mean for this density.
c. A electronic supply firm wants to "burn in" each device T seconds before selling so that 99% of the failures happen during the "burn in" period. Find T.