MAC 2313 Calculus 3 **Test 1** 28 Sep 2002

Show **ALL** work for credit; be neat; and use only **ONE** side of each page of paper. Do **NOT** write on this page. Calculators can be used for graphing and calculating only. Give exact answers when possible.

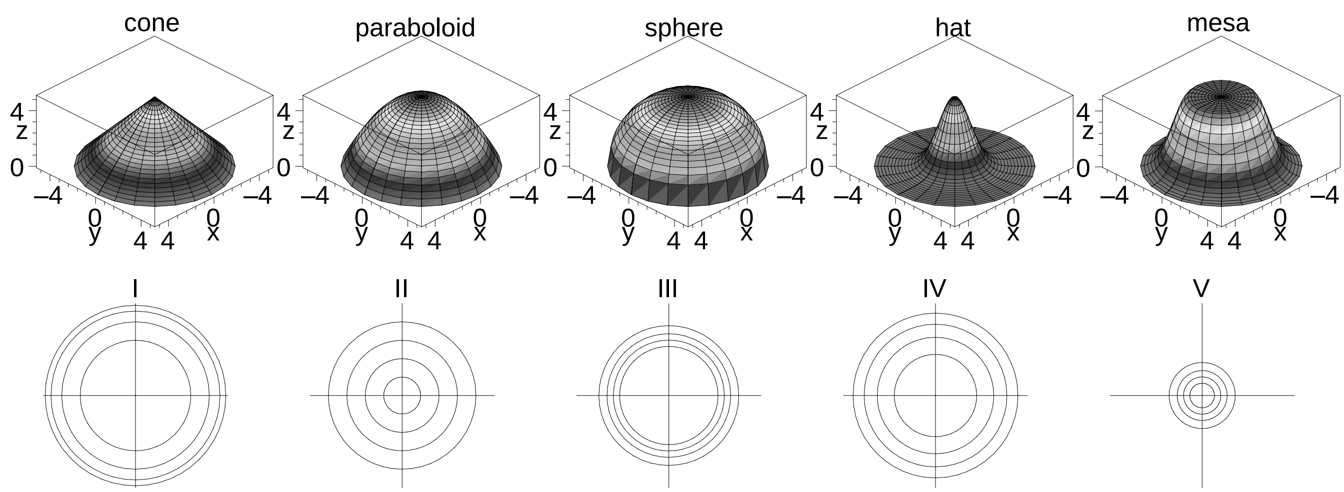
1. Find the equation of the plane parallel to the plane $3x - 4y - 6z = 21$ and passing through the point (-3,1, 2) and find the distance between the two parallel planes.

2. Find the equation of the plane through the points (-1, 1, -1), (1, -1, 2) and (4, 0, 3).

3. A woman walks due west on the deck of a huge airship at 1 km/h. The ship is moving north at a speed of 8 km/h and climbing at a rate of 4 km/h Find the velocity vector of the women relative to the ground. Find her speed and a unit vector in the direction of the velocity. (The x-axis points East, the y-axis points North, and the z-axis points up.)

4. Find the center and radius of the sphere S given by the equation $x^2 + y^2 + z^2 + 2x + 8y - 4z = 28$. The graph of S intersects the xz-plane in a circle, what is its equation, its center and its radius?

5. Match the plot3ds to the contourplots. Each contourplot plots the four contours $z = 1, 2, 3$ and 4 and each 3d plot is over the disk $x^2 + y^2 \leq 25$.



6. Determine if the lines L_1 and L_2 are parallel, skew or intersecting. If they intersect, find the point of intersection.
L_1: $x = 2 + t, y = 2 - t, z = 5 + 3t$
L_2: $x = 1 - s, y = 1 + 2s, z = -6 + s$

7. Find the parametric equation of the line through the points $P(3, 2, 8)$ and $Q(4, 4, -4)$ and find the two points where it intersects the elliptical paraboloid $z = x^2 + y^2$.

8. Write $\mathbf{a} = \langle 3, -1, 5 \rangle$ as the sum of two vectors, one parallel (say $\mathbf{v}$), and one perpenducular (say $\mathbf{w}$) to $\mathbf{b} = \langle 1, 1, 1 \rangle$. Draw all four of these vectors **CAREFULLY** in three space. (You can put the vectors on the same graph or on several graphs on the same page, which ever you like. But be sure to include the **SCALE** by having tickmarks on each axis.)

9. Check your trusty calculator and make sure it is in radian mode. Let $\mathbf{r}(t) = \langle t\cos(t), t\sin(t) \rangle$ for $0 \leq t \leq 2\pi$
 a. Find the velocity of $\mathbf{r}(t)$.
 b. Plot the curve for $0 \leq t \leq 2\pi$
 c. Find the speed and write an integral which will give the arclength of the curve for $0 \leq t \leq 2\pi$
 d. Find the exact arclength.
 e. Find a numerical approximation to the arclength.

Problem 10 is on the other side.

10. For questions below list all of equations A–H (see below) that satisfy the given condition, if there are none that satisfy condition then say "none". [Hint: Sometimes it is easier to say all but "these", then to list the ones that do.]

a. Which are hyperboloids?
b. Which are cylinders?
c. Which contain the origin?
d. Which are unbounded?
e. Which intersect the y-axis?

The list of equations:

A. $x^2 + 4y^2 + 9z^2 = 1$
B. $9x^2 + 4y^2 + z^2 = 1$
C. $x^2 - y^2 + z^2 = 1$
D. $-x^2 + y^2 - z^2 = 1$
E. $y^2 = 2x^2 + z^2$
F. $y = x^2 + 2z^2$
G. $x^2 + 2z^2 = 1$
H. $y = x^2 - z^2$