

**Directions:** Show **ALL** work for credit; Give **EXACT** answers when possible; Start each problem on a **SEPARATE** page; Use only **ONE** side of each page; Be neat; Leave margins on the left and top for the **STAPLE**; Calculators can be used for graphing and calculating only; Nothing written on this page will be graded;

1. Sketch the region of integration and then change the order of integration of

$$\int_0^1 \int_{\sqrt{y}}^1 f(x, y) dx dy$$

$$\int_0^1 \int_0^{x^2} f(x, y) dy dx$$

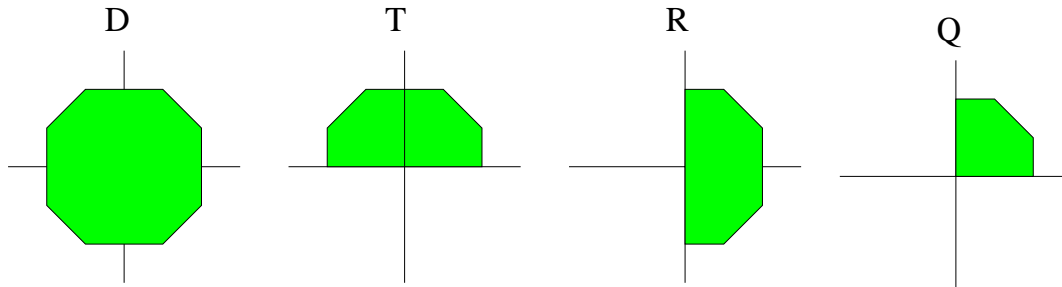
2. Let  $A(0, 0)$ ,  $B(5, 0)$  and  $C(3, 4)$ . Find the coordinates of the point  $P$  in the  $xy$ -plane so the sum of the distance squares  $\|\vec{PA}\|^2 + \|\vec{PB}\|^2 + \|\vec{PC}\|^2$  is minimum.

The function to minimize is  $f(x, y) = (x - 0)^2 + (y - 0)^2 + (x - 5)^2 + (y - 0)^2 + (x - 3)^2 + (y - 4)^2$ . There is a global minimum, which has to be a local minimum, which has to occur at a critical point. Looking  $f_x = 2x + 2(x - 5) + 2(x - 3) = 0$  when  $3x - 8 = 0$  and  $f_y = 2y + 2y + 2(y - 4) = 0$  when  $3y - 4 = 0$  so the only critical is at  $(8/3, 4/3)$  which must be the coordinates of  $P$ .

3. The regions pictured below are inside the unit circle, decide if the following double integrals are positive, negative or zero.

$$A = \iint_D x^2 + y^2 dA \quad B = \iint_T 5y dA \quad C = \iint_T 5x dA \quad D = \iint_R 5x dA \quad E = \iint_Q 0 dA$$

$$F = \iint_T -\sin(x) dA \quad G = \iint_T -\cos(x) dA \quad H = \iint_T y^3 - y dA \quad I = \iint_R y^3 - y dA \quad J = \iint_R x + y^2 dA$$



A+ B+ C0 D+ E0  
F0 G- H- I0 J+

4. Use your TI-89 to find all the critical points of the function  $f(x, y) = 8y^3 + 12x^2 - 24xy$ , then show how you would obtain these critical points by hand. Classify these local extrema by filling out a table like the one below, with a separate line for each critical point.

$(x, y)$	$f_{xx}$	$f_{yy}$	$f_{xy}$	big D	Classification
?	?	?	?	?	?

The calculator gives two critical points. By hand we find  $f_x = 24x - 24y$  and  $f_y = 24y^2 - 24x$ . Setting  $f_x = 0$  yields  $y = x$  and plugging into  $f_y = 0$  yields  $y^2 - y = y(y - 1) = 0$ , so we have critical points at  $(0, 0)$  and  $(1, 1)$ . We need  $f_{xx} = 24$ ,  $f_{yy} = 48y$ , and  $f_{xy} = -24$ . The table becomes

$(x, y)$	$f_{xx}$	$f_{yy}$	$f_{xy}$	big D	Classification
$(0, 0)$	24	0	-24	-576	Saddle
$(1, 1)$	24	48	-24	576	Local Minimum

5. Use Lagrange multipliers to find the maximum and minimum **VALUES** of  $f(x, y) = x^2 - y$  subject to the constraint that  $x^2 + y^2 = 4$ .

Let  $F(x, y, \lambda) = x^2 - y - \lambda(x^2 + y^2 - 4)$ .  $F_x = 2x - 2\lambda x$ ,  $F_y = -1 - 2\lambda y$ , and  $F_\lambda = -(x^2 + y^2 - 4)$ . Setting  $F_x = 0$  yields  $x(1 - \lambda) = 0$  so  $x = 0$  or  $\lambda = 1$ . Setting  $F_y = 0$  yields  $y = -1/(2\lambda)$ . Keying off  $F_x = 0$  if  $x = 0$ , then plugging in  $x^2 + y^2 = 4$  yields  $y = \pm 2$  and two points to check. On the other hand if  $\lambda = 1$ , then the  $f_y$  equation says  $y = -1/2$ . Plugging this  $y$  into the constrain yields  $x = \pm\sqrt{15}/2$ . Checking values:

$$f(0, 2) = -2$$

$$f(0, -2) = 2$$

$$f(\sqrt{15}/2, -1/2) = 17/4$$

$$f(-\sqrt{15}/2, -1/2) = 17/4$$

so  $17/4$  is the maximum value and  $-2$  is the minimum value.