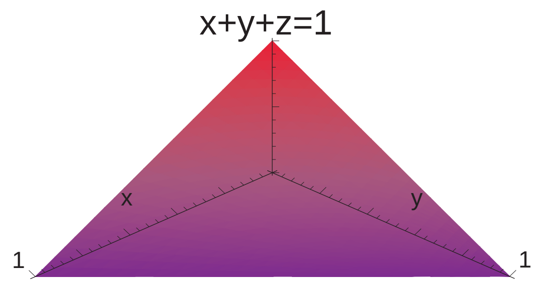
MAC 2313 Calculus 3 **Mini-Test 5** 24 Mar 2004

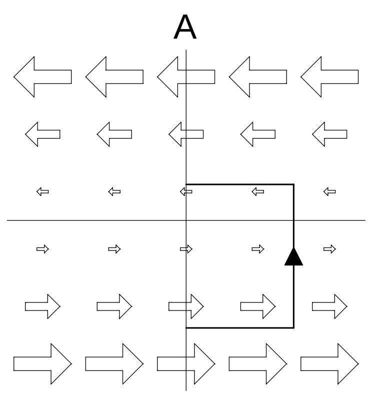
Directions: Show **ALL** work for credit; Give **EXACT** answers when possible; Start each problem on a **SEPARATE** page; Use only **ONE** side of each page; Be neat; Leave margins on the left and top for the **STAPLE**; Calculators can be used for graphing and calculating only; Nothing written on this page will be graded;

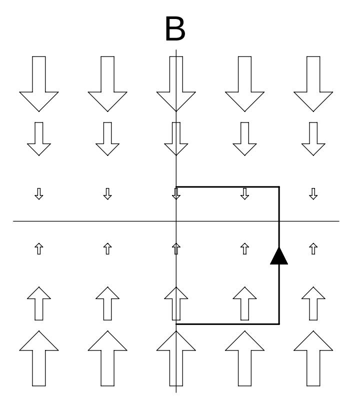
1. Write a triple integral which will give the mass of the solid (see below) bounded by the xy-plane, yz-plane, xz-plane and the plane $x+y+z=1$, if the density of the solid is given by $\delta(x,y,z) = y+2z$. Evaluate the integral using your TI-89.



$$\int_0^1 \int_0^{1-x} \int_0^{1-x-y} y + 2z \, dz \, dy \, dx = \frac{1}{8}$$

2. Find formulas for the two vector fields below. (There are many possible answers.) Decide if the line integrals over the given curves will be positive negative or zero in each plot.





A. is $\langle -y, 0\rangle$ and the line integral is positive. B. is $\langle 0, -y\rangle$ and the line integral is positive.

3. Parametrized surfaces where each of s and t is one of the coordinates x, y, z, r, θ, ρ, or ϕ.

   (a) Write the paraboloid $z = f(x, y) = x^2 + y^2$ for $0 \le x \le 1, 0 \le y \le 1$ as a parametrized surface $\vec{r}(s, t) = \langle x(s, t), y(s, t), z(s, t)\rangle$. Give the ranges for s and t.

   (b) Write the cone $z = r$ for $0 \le r \le 5$ as a parametrized surface $\vec{r}(s, t) = \langle x(s, t), y(s, t), z(s, t)\rangle$. Give the ranges for s and t.

   (c) Write the portion of the sphere $\rho = 4$ for $-2 \le z \le 2, \pi/4 \le \theta \le 3\pi/4$ as a parametrized surface $\vec{r}(s, t) = \langle x(s, t), y(s, t), z(s, t)\rangle$. Give the ranges for s and t.

(a) is $\vec{r}(s, t) = \left\langle s, t, s^2 + t^2\right\rangle$. Ranges $0 \le s \le 1, 0 \le t \le 1$.
(b) is $\vec{r}(s, t) = \langle s\cos t, s\sin t, s\rangle$. Ranges $0 \le s \le 5, 0 \le t \le 2\pi$.
(c) is $\vec{r}(s, t) = \langle 4\sin s\cos t, 4\sin s\sin t, 4\cos s\rangle$. Ranges $\pi/3 \le s \le 2\pi/3, \pi/4 \le t \le 3\pi/4$.
Alternately, (c) is $\vec{r}(s, t) = \left\langle s, \sqrt{16 - s^2 - t^2}, t\right\rangle$. Ranges $-2\sqrt{2} \le s \le 2\sqrt{2}, -2 \le t \le 2$.

4. Let T be the triangle in the xy-plane between $x = 0$ and $x = 5$ above the line $y = -x$ and below the line $y = x$. Write a double integral in polar coordinates which the mass of T if it has a density function $\delta(r, \theta) = 1/r + \cos^2\theta$. Do NOT evaluate.

$$\int_{-\pi/4}^{\pi/4}\int_0^{5/\cos\theta} 1 + r\cos^2\theta \, dr \, d\theta$$

5. Sketch the region (include drawings of the "shadow" in the xy-plane and a 2D z vs r plot ($r \ge 0$)) for integral below. Rewrite the triple integeral in spherical coordinates and evaluate it by hand.

$$\int_{-3}^{3}\int_0^{\sqrt{9-x^2}}\int_0^{\sqrt{9-x^2-y^2}} \sqrt{x^2 + y^2}\, dz\, dy\, dx$$

$$= \int_0^{\pi}\int_0^{\pi/2}\int_0^{3} (\rho\sin\phi)\rho^2\sin\phi \, d\rho\, d\phi\, d\theta$$

$$= \pi\int_0^{\pi/2} \frac{\rho^4}{4}\Big|_0^3 \sin^2\phi \, d\phi = \frac{81\pi}{4}\frac{1}{2}\frac{\pi}{2} = \frac{81\pi^2}{16}$$