Practice Mini-Test 4 – Calculus 3 – Spring 04

1. T2#7 F03 Classify local extrema. [Compare T2#9 S03 T2#9 F02 T2#10 S02] Use your TI-89 to find all the critical points of the function $f(x, y) = x^3 - 3xy + y^3$, then show how you would obtain these critical points by hand. Classify these local extrema by filling out a table like the one below, with a separate line for each critical point.

(x, y)	f_{xx}	f_{yy}	f_{xy}	big D	Classification
?	?	?	?	?	?

The TI-89 says there are two roots. We need some derivatives $f_x = 3x^2 - 3y$, $f_y = -3x + 3y^2$, $f_{xx} = 6x$, $f_{yy} = 6y$ and $f_{xy} = -3$. Setting $f_x = 0 = f_y$ throwing away the common factor of 3 we have two equations $y = x^2$ and $x = y^2$ which imply $x = x^4$ or $x^4 - x = x(x^3 - 1) = x(x-1)(x^2 + x + 1)$. The quadratic term has only complex roots so we have $x = 0$ with $y = 0^2 = 0$ and $x = 1$ with $y = 1^2 = 1$. Our final table is below.

(x, y)	f_{xx}	f_{yy}	f_{xy}	big D	Classification
$(0, 0)$	0	0	-3	-9	saddle point
$(1, 1)$	6	6	-3	27	local min

2. T2#10 F02 Lagrange multipliers. [Compare T2#10 F03 T2#8 S03] Use your TI-89 to plot the $z = 1$ contour of the function $z = g(x, y) = x^2 + xy + y^2$. On the same graph, plot some contour lines for $f(x, y) = x + y$. Use Lagrange Multipliers to find the maximum and minimum **VALUES** for $f(x, y)$ on the constraint $g(x, y) = 1$.

The contour is an ellipse with major axes along the line $y = -x$. Let $F(x, y, \lambda) = x + y - \lambda(x^2 + xy + y^2 - 1)$. $F_x = 1 - \lambda(2x + y)$ $F_y = 1 - \lambda(x + 2y)$ and setting each equal to zero and solving for λ "works" in this case to give $x = y$. Pluging into the constrain gives $3x^2 = 1$ or $x = \pm 1/\sqrt{3}$ and we have two points to check. Now $f(1/\sqrt{3}, 1/\sqrt{3}) = 2/\sqrt{3}$ $f(-1/\sqrt{3}, -1/\sqrt{3}) = -2/\sqrt{3}$. So Maximum value is $2/\sqrt{3}$ and minimum value is $-2/\sqrt{3}$

3. T3#4 S03 Change the order [Compare T2#8 F02 T3#6 S02] Sketch the region of integration and reverse the order of integration for

$$\int_0^2 \int_{x^2}^{2x} dy\, dx$$

[You do **NOT** have to evaluate the integrals, but using the TI-89 to evaluate both integrals would be a way of checking your answer.]

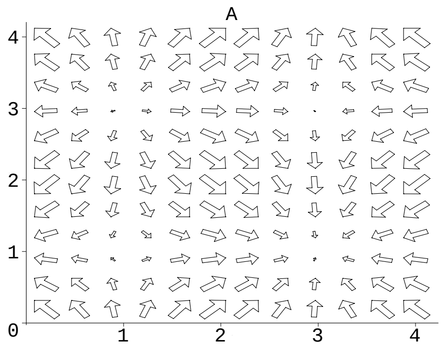
Use the plot function of your calculator with a window of x from 0 to 2 and for y use 0 to 4 plot both $y = 2x$ and $y = x^2$. The region is below the line and above the parabola.

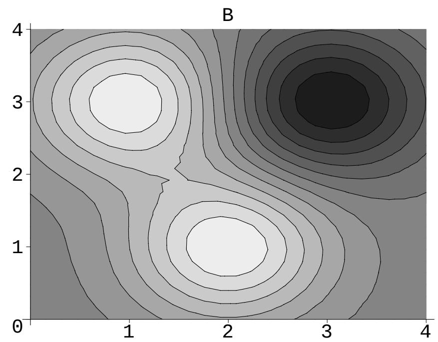
$$\int_0^4 \int_{y/2}^{\sqrt{y}} dx\, dy$$

4. T2#10 S03 Optimization. [Compare T2#9 F96] Find the coordinates of the **POINT** closest to the point $P(1, 1, 0)$ which is both on the surface $z^2 = x^2 + y^2$ AND is in the first octant.

Minimize the distance squared $f(x, y, z) = (x-1)^2 + (y-1)^2 + (z-0)^2$ on the surface $z^2 = x^2 + y^2$. The z^2 term in the constrain is very nice and the problem becomes minimize $g(x, y) = (x-1)^2 + (y-1)^2 + x^2 + y^2$. There must be a global min, which has to occur at a local min. Looking for critical points: $g_x = 2(x-1) + 2x = 0$ or $x = 1/2$ and the same for y so the point in the first octant is $(1/2, 1/2, 1/\sqrt{2})$

5. T2#4 F02 Local Extremes from graphs [Compare T2#9c S02 T2#5] The graph A is a plot of ∇f, the gradient of f and the graph B is a contourplot of g. (Light regions have higher values than dark regions.] Find the co-ordinates of all extrema of f and g and **LABEL** them as either local minimums, local maximums or saddle points.





The function f has a local minimum at $(1,3)$, a local maximum at $(3,1)$ and saddles at $(1,1)$ and $(3,3)$. The function g has a local minimum at $(3,3)$, local maximums at $(1,3)$ and $(2,1)$ and a saddle at $(1.5,2)$.

6. Positive, Negative or Zero. Let $D = \{(x,y) : x^2 + y^2 \le 1\}$, $T = \{(x,y) : y \ge 0, x^2 + y^2 \le 1\}$, $R = \{(x,y) : x \ge 0, x^2 + y^2 \le 1\}$ and $Q = \{(x,y) : x \ge 0, y \ge 0, x^2 + y^2 \le 1\}$. Determine if the following are positive negative or zero.

$\iint_D x\, dA$	$\iint_T x\, dA$	$\iint_R x\, dA$	$\iint_Q x\, dA$
$\iint_D x^2 + y^2\, dA$	$\iint_T x^2 + y^2\, dA$	$\iint_R x^2 + y^2\, dA$	$\iint_Q x^2 + y^2\, dA$
$\iint_D \sin y\, dA$	$\iint_T \sin y\, dA$	$\iint_R \sin y\, dA$	$\iint_Q \sin y\, dA$
$\iint_D xy\, dA$	$\iint_T xy\, dA$	$\iint_R xy\, dA$	$\iint_Q xy\, dA$

0	0	+	+
+	+	+	+
0	+	0	+
0	0	0	+

7. T2#6 F00 Graphs from local extrema. The function $f(x,y)$ has local maximums at $(1,0)$ and $(-1,0)$, local minimums at $(0,1)$ and $(0,-1)$ and one saddle point at the orgin.

   (a) Sketch a possible contour graph for f.

   (b) Sketch a possible graph for ∇f.

Check out $\sin(\pi(x+y)/2)\sin(\pi(x-y)/2)$ in the range $-1.5 \le x \le 1.5$ and $-1.5 \le y \le 1.5$