HW2

1.26: Hints given in class.

1.30: Almost too easy with loops. The Handshaking Lemma saids $\sum_{i=1}^{p} d_i$ must be even when $d_1, d_2, \ldots d_p$ is a degree sequence. Conversely, let $d_1, d_2, \ldots d_p$ be given with $\sum_{i=1}^{p} d_i$ even. Construct G as follows. Let $v_1, v_2, \ldots v_p$ be the vertices of G. If d_i is even add $d_i/2$ loops at v_i. If d_i is odd add $(d_i - 1)/2$ loops at v_i. Since the number of odd $d_i's$ is even, pair up the odd $d_i's$ and add an edge from v_i to v_j if d_i was paired to d_j. G has the required degree sequence.

1.30 *By Induction.* (On $n = \sum_{i=1}^{p} d_i$) If $n = 0$ use the empty graph. Assume true for n, that is if $n \sum_{i=1}^{p} d_i$ then there is a multigraph G with the degree sequence $d_1, d_2, \ldots d_p$. Let $d_1 \geq d_2 \geq \cdots \geq d_p$ be a sequence with $\sum_{i=1}^{p} d_i = n+2$. If $d_1 = 1$, then $d_2 = 1$, and the sequence $d_1 - 1, d_2 - 1, d_3, \ldots d_p$ is the degree sequence of a graph G with vertices $v_1, v_2, ...v_p$. Adding an edge v_1v_2 to G yields a multigraph with the required degree sequnce. Otherwise $d_1 \geq 2$, apply the induction hypothesis to $d_1 - 2, d_2, \cdots, d_p$ and then at a loop at v_1.

1.31: Let G be a (p, q)-graph with vertices v_i of degree d_i. The line graph $L(G)$ has q vertices, one for each edge G. The degree of the vertex v in G is the number of edges with one end at v. There is an edge in $L(G)$ for every pair of edges in G which have one end at v. Thus the vertex v with degree d results in $(d - 1)d/2$ edges in $L(G)$. [Note this formula also works when $d = 0, 1$.] So the number of edges in $L(G) = \sum_{v \in V(G)} deg(v)(deg(v) - 1)/2$.