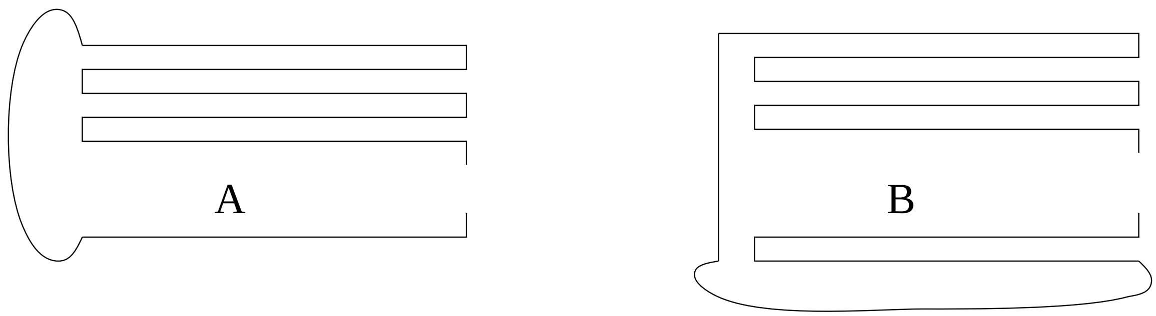
HW9

5.13: To show $G \times H$ is Hamiltonian when both G and H are Hamiltonian it suffices to show $C_n \times C_m$ is Hamiltonian. This is best proved by a picture (the figure below.) Figure A is when m (the y or second coordinate) is even and Figure B is for when m is odd.



5.15: To show Q_n is Hamiltonian it suffices to show $K_2 \times C_n$ is Hamiltonian. (This yields an inductive prove since Q_2 is C_4 and $Q_{n+1} = K_2 \times Q_n$.) And his is really just Figure A (above) again.

5.44: The (6,3)-cage is drawn below. To show it is unique, let G be a (6,3)-cage and pick any vertex and call it 1. Being 3-regular gives vertices 2, 3 and 4. Being 3-regular gives vertices 5, 6, 7, 8, 9, and 10 and the girth of 6 requires all of these are distinct. The 3-regularity of 5 and 6, yield the vertices 11, 12, 13, and 14 the girth requirement at 2 requires that these are all distinct. Since the graph below is a (6,3)-cage, any such graph must have 14 vertices. Thus to complete the degrees of 7 and 8 (and 9 and 10) require these to be connected to 11, 12, 13, and 14. Vertex 7 can't be connected to both 11 and 12 (or both 13 and 14) or 7-11-5-12-7 would have girth 5. Renaming we may assume 7 is connected to 11 and 13 and 8 is connected to 12 and 14. Two similar agreements show 9 is not connected to both 11 and 12 and 9 is not connect to both 11 and 13. So we can assume 9 is connected to 12 and 13 and 10 is connected to 11 and 14 (or we can swap them.) So it must be the graph below.