MAC 2311 Calculus 1 **Test 2** 25 Feb 1999

Show **ALL** work for credit; be neat; and use only **ONE** side of each page of paper. Do **NOT** write on this page. Calculators can be used for graphing and calculating only. Give exact answers when possible.

1. Find the area under the first arch (from 0 to $\sqrt{\pi}$) of $y = \sin(x^2)$ (State the numerical method you used.)

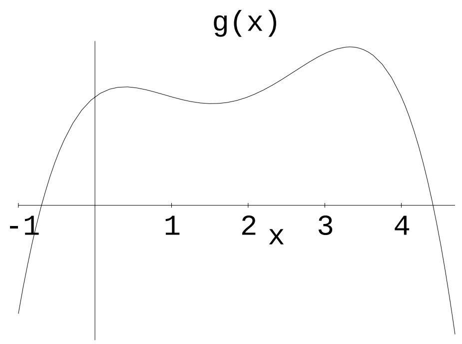
2. Find an equation for the tangent line to $f(x) = x^2$ at $x = 3$. Plot $f(x)$ and this tangent line.

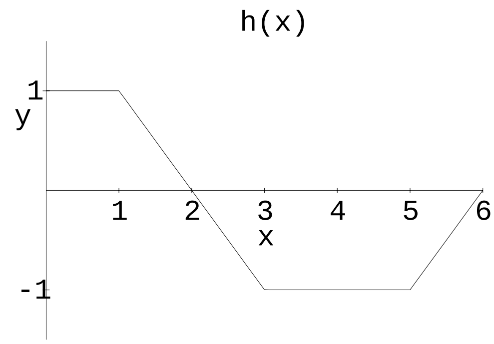
3. A car going 80 ft/sec (about 55 mph) brakes to a stop in 8 seconds. Its velocity is recorded every 2 seconds and is given in the following table
(a) Give your best estimate of the distance traveled by the car during the 8 seconds.
(b) To estimate the distanced traveled accurate to within 20 feet, how often should you record the velocity.

t(seconds)	0	2	4	6	8
$v(t)$(ft/sec)	80	52	28	10	0

4. For an even function f
(a) Suppose you know $\int_0^2 f(x)dx$. What is $\int_{-2}^2 f(x)dx$?
(b) Suppose you know $\int_0^5 f(x)dx$, and $\int_2^5 f(x)dx$. What is $\int_0^2 f(x)dx$?
(c) Suppose you know $\int_{-2}^5 f(x)dx$, and $\int_{-2}^2 f(x)dx$. What is $\int_0^5 f(x)dx$?

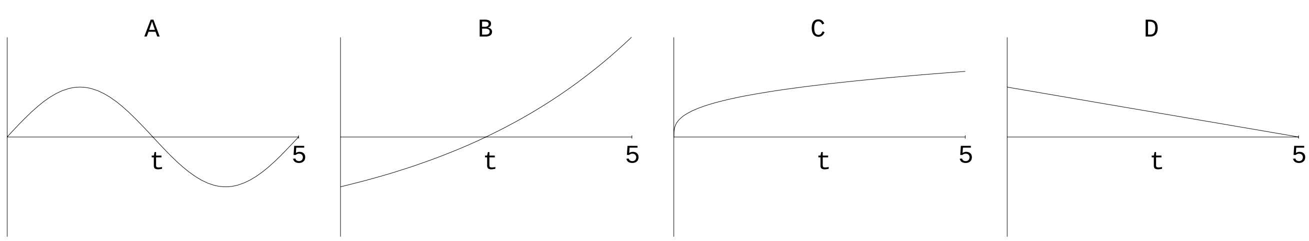
5. Sketch the graph of the derivative to the function $g(x)$ in the graph below. (You might want to trace $g(x)$ onto your answer sheet.)





6. The graph above plots the function $h(x)$. If $H' = h$ and $H(0) = 0$, find $H(b)$ for $b = 1, 2, 3, 4, 5, 6$.

7. Each of the graphs A-D below shows the position of a particle moving along the x-axis as a function of time, $0 \leq t \leq 5$. (The x-axis is vertical.) The vertical scales of the graphs are the same. During this time interval, which particle has
(a) Constant velocity?
(b) The greatest initial velocity?
(c) The greatest average velocity?
(d) Zero average velocity?
(e) Positive acceleration throughout?



There is more test on the other side.

Welcome to test two side two.

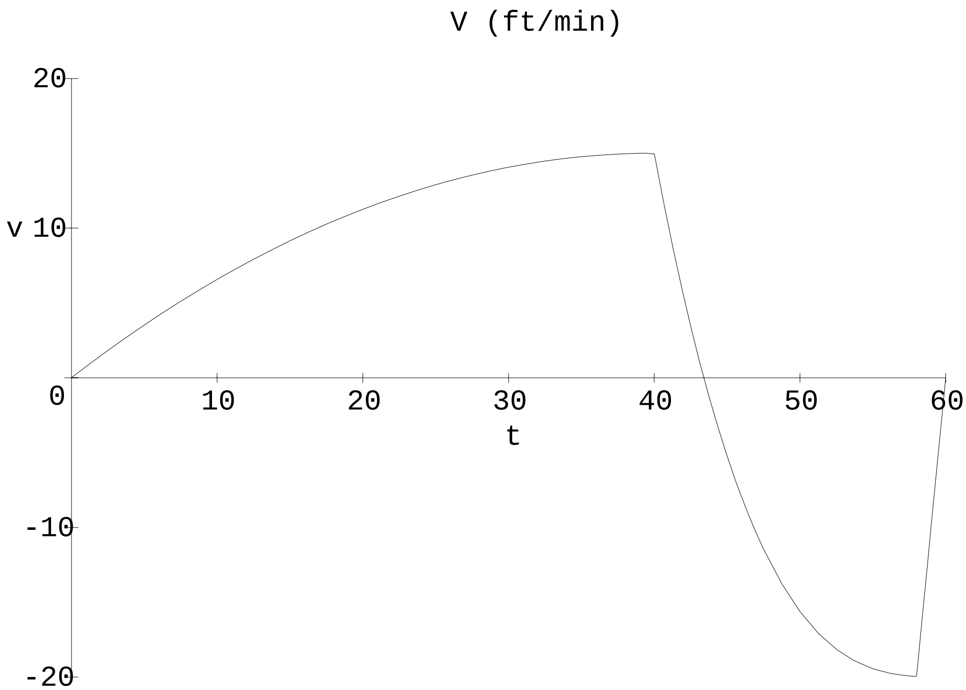
8. Find the derivative of $f(x) = 1/x$ algebraically.

9. Students were asked to evaluate $f'(4)$ from the following table.

x	1	2	3	4	5	6
$f(x)$	4.2	4.1	4.2	4.5	5.0	5.7

- Student A estimated the derivative as $f'(4) \approx \frac{f(5)-f(4)}{5-4} = 0.5$.
- Student B estimated the derivative as $f'(4) \approx \frac{f(4)-f(3)}{4-3} = 0.3$.
- Student C suggested that they should split the difference and estimate the average of these two results, that is, $f'(4) \approx \frac{1}{2}(0.5 + 0.3) = 0.4$.

(a) Sketch the graph of $f(x)$ and indicate how these three estimates are represented on the graph.
(b) Explain which answer is likely to be best.
(c) Use Students C's method to find an algebraic formula which approximates $f'(x)$ using increments of size h.



10. The Montgolfier brothers (Joseph and Etienne) were eighteenth-century pioneers in the field of hot-air balloning. Had they had the appropriate instruments, they might have left us a record of one of their early experiments, like the one show above. The graph show their vertical velocity, v, with upward as positive.
(a) Over what intervals is the acceleration positive? Negative?
(b) What was the greatest altitude achieved, and at what time?
(c) At what time was the deceleration the greatest?
(d) What might have happend during the this flight to explain the answer to part (c)?
(e) This particular flight ended on top of a hill. How do you know that it did and what was the height of the hill above the starting point?