MAP 2302 Diff-E-Qs **Lab 2** Summer 2005

Directions: Use only **ONE** side of each page, use ink and a staple.

This lab is about chaos, in the sense of the butterfly effect. The great ODE examples of choas are too late in the book. So this is a discrete system which is described in Section 2.9. The equation is

$$x_{n+1} = \lambda x_n(1 - x_n)$$

we are just going to explore this and not try to explain it. Remember to keep $x_n \in [0, 1]$ λ must be restricted to $0 \leq \lambda \leq 4$.

We want to explore what happens as we change λ. Remember to explain how you used your technology to obtain the numbers. Also do not forget that clarity/presidentation are part of the grade.

1. The small λ region. In the limit

$$x_\infty = \lambda x_\infty(1 - x_\infty)$$

which has two roots 0 and $\frac{\lambda-1}{\lambda}$ Numerially verfy this using your choice of technology by computing x_{100} when $x_0 = 0.5$ for various λ in the table below:

λ	$(\lambda - 1)/\lambda$	x_{100}	x_{101}
0.5	?	?	?
1.0	?	?	?
1.5	?	?	?
2.0	?	?	?
2.5	?	?	?
3.0	?	?	?
3.5	?	?	?
4.0	?	?	?

Surprises? Explain them away (Hint what happens if the initial $x_0 \neq 0.5$)

2. The period two region $3 < \lambda < 3.449$ theory says that $x_{n+2} \approx x_n$ if x is a solution to

$$x = \lambda\big(\lambda * x(1 - x)\big)\big(1 - \big(\lambda * x * (1 - x)\big)\big) = \lambda^2 x(1 - x - \lambda x + 2\lambda x^2 - \lambda x^3)$$

which has four roots, the two from the case above and $A \pm B$ where

$$A = \frac{\lambda + 1}{2\lambda} \qquad B = \frac{\sqrt{\lambda^2 - 2\lambda - 3}}{2\lambda}$$

Fill out the table below, assumeing $x_0 = 0.5$

λ	$A + B$	$A - B$	x_{100}	x_{101}	x_{102}
3.1	?	?	?	?	?
3.25	?	?	?	?	?
3.4	?	?	?	?	?
3.5	?	?	?	?	?

3. According to the text, the solution changes from period 2 to period 4 around $\lambda = 3.449$, do a numerical experiment to verify this.

4. Chaos, for $\lambda = 3.65$ plot two solutions on the same graph one with $x_0 = 0.5$ and the other which differs by a butterfly $x_0 = 0.501$. When do the two plots clearly "fly off" in different directions.