

Show $\mathcal{P}(X)$ has is not an ordered field, but it is a field with a partial order given by $A \leq B \iff A \subseteq B$ when $A \cdot B$ is defined as $A \cap B$ and $A + B$ is defined as the symmetric difference $(A \setminus B) \cup (B \setminus A)$, it satisfies all the axioms of a complete order field, but the law of trichotomy.

Note that $A \cap B \cap C = \{x \in X | x \in A, x \in B \text{ and } x \in C\}$ and $A + B + C = \{x \in X | x \text{ is in an odd number of } A, B, C\}$ which makes the associative law true.

Note that any collection \mathcal{A} of subsets of X is always bounded above by X and below by \emptyset , the sup is $\cup \mathcal{A}$ and the inf is $\cap \mathcal{A}$