E is for Evil

This welcome is brought to you
by Abel's opinion on divergent series,
Euler's cleverness,
and Walt Disney

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The talk is a slide show. The slides are framed in yellow rectangles. The quotation that follows, is what might have been said while the audience was looking at the slide. The blue comments like this one were added later and not part of the welcome. The title frame above was not the original.

Classic Origin of Good and Evil

This is the garden of eden complete with snake and apple tree.

My wife did jewerly before she retired. But not this style, she worked in brass and later polymer clay.

Euler Sums a Divergent Series

$$\sum_{n=0}^{\infty}(-1)^n n! = 0.59637255\ldots$$

Abel

Divergent series are in general the the work of the devil and it is shameful to base any demostration whatever on them

Abel's famous statement on diverenge series being the work of the devil. And Euler who somehow manages to get a number out of a series whose terms blow up rapidly.

Of course, there are many ways to deal with divergent series. G.H. Hardy has a famous book on 'Divergent Series' published in 1956.

How Some Advisors View the Math Department

https://www.theguardian.com/books/2016/feb/17/the-dark-side-of-the-soul-an-insiders-guide-to-the-wel

That is what I said.

new stuff

Advisors (other than Esther) are not your friend

- Do not reply to email from students wanting to add your class just forward them to advisor@math.fsu.edu

That is what I said.

new stuff

The Evils of Student Email

You do not have to reply to student email. In particular, you don't have to repeat a lecture they missed. Nor do you have to hold office hours by email. Nor are you expected to continuous check you email for student questions. The recommendation is that you let email compost before answering.

While email is good for 'I'm sick' it isn't very good for conversations that have a level of depth.

Disney Again, You need a Crystal Ball

similar with different background `https://comics.ha.com/itm/animation-art/production-cel/sleeping-beauty-ma` `a/7148-95166.s`

This slide was used as a backdrop on ADA students. The crystall ball reference was to highlight the need to have a conversation with these students in your office to find out what they are requesting. Often the math department has guidelines for TAs, passing unusal requests to particular faculty.

Euler

$$f(x) = \sum_{n=0}^{\infty} (-1)^n n! x^n = 1 - 1!x + 2!x^2 + \cdots$$

$$g(x) = xf(x) = \sum_{n=0}^{\infty} (-1)^n n! x^{n+1} = x - 1!x^2 + 2!x^3 + \cdots$$

$$g'(x) = \sum_{n=0}^{\infty} (-1)^n (n+1)! x^n = 1! - 2!x + 3!x^2 + \cdots$$

$$x^2 g'(x) = \sum_{n=0}^{\infty} (-1)^n (n+1)! x^{n+2} = 1!x^2 - 2!x^3 + 3!x^4 + \cdots$$

$$x^2 g'(x) + g(x) = x$$

Solve the ODE

$$g' + (1/x^2)g = 1/x$$

Note that the power series for $f(x)$ only convergences for $x = 0$. So Abel would the devil is at work, and the first four lines are not to be trusted. Euler does anyway, and gets an ODE for $g(x)$.

Perhaps the question might be, is there any other ODE which could have been generated? Alternately, we are in abstraction of formal power series and evaluation maps or linear functionals. Stuff that came long after Euler. Asymptotic expansions.

Solve the ODE

Solve the ODE

$$g' + (1/x^2)g = 1/x$$

$$(g(x)\exp(-1/x))' = \exp(-1/x)/x$$

$$g(x)\exp(-1/x) = C + \int \exp(-1/x)/x\,dx \quad \text{eventually } C = 0$$

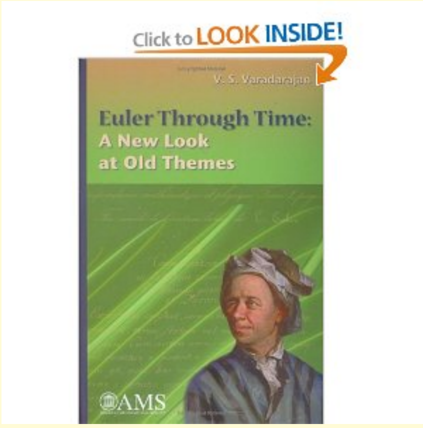
$$g(x) = \exp(1/x) \int_0^x \exp(-1/x)/x\,dx$$

and Euler numerically computed the integral for $x = 1$

We can solve this ODE using an integrating factor of $\exp(1/x)$.

new stuff

A reference



That is what I said.

new stuff

Picture sources

x. picture is from