Spring 2013 Welcome

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Talking points

- The square root of two is irrational.
- The right way to do email. You don't have to answer it.
- Send them up to the Coordinators or me.
- You are the math department, most students do not see a research instructor.

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All new software

myFSU Student Central

- Degree programs called Careers
- Degrees shorten to BACH DOCT MASTR
- Majors codes disappear: 116811, 11=Arts and Sciences, 68=Mathematics, 11=sequence number for ACM.The new way: RC_PLN_UG_Appld/Cmptatn Math and RC_PLN_GD_Mathematics (Appld)

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The Pythagorean: Hippasus of Metapontum

Infinite Descent, Escher's Waterfall

Suppose $\sqrt{2} = a/b$ in lowest terms

Let $a_n = (\sqrt{2}-1)^n a$ and $b_n = (\sqrt{2}-1)^n b$. Since $1 < \sqrt{2} < 2$, $0 < \sqrt{2}-1 < 1$ and

$$a_0 > a_1 > a_2 > \cdots > 0, \qquad b_0 > b_1 > b_2 > \cdots > 0$$

$$\frac{a_n}{b_n} = \frac{a}{b} = \sqrt{2} \implies \sqrt{2}b_n = a_n \qquad \sqrt{2}a_n = 2b_n$$

$$a_{n+1} = (\sqrt{2}-1)a_n = 2b_n - a_n, \qquad b_{n+1} = (\sqrt{2}-1)b_n = a_n - b_n$$

Both sequences are strictly decreasing sequences of positive integers.

Suppose $\sqrt{2} = a/b$ in lowest terms

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It Ain’t Me Babe

For integers a and b, $|\sqrt{2} - a/b| > 1/3b^2$

Case 1: $0 \leq a/b \leq 3/2$,

$$|\sqrt{2} - a/b| = |\sqrt{2} - a/b| \frac{|\sqrt{2} + a/b|}{|\sqrt{2} + a/b|} = \frac{|2 - a^2/b^2|}{\sqrt{2} + a/b} > \frac{|2b^2 - a^2|}{3b^2} \geq \frac{1}{3b^2}$$

Case 2: $a/b > 3/2$,
$|\sqrt{2} - a/b| > |3/2 - a/b| = |3b - 2a|/|2b| > 1/3b^2$

Case 3: $a/b < 0, |\sqrt{2} - a/b| > 1 > 1/3b^2$

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The advisor is not your friend

Theorem. If p and q are relatively prime and p/q is a root of $a_n x^n + a_{n-1} x^{n-1} + \dots a_1 x + a_0$ with integer coefficients then $p \mid a_0$ and $q \mid a_n$.

Proof: Substitute $x = p/q$ and multiply by q^n

$$\overbrace{a_n p^n + \underbrace{a_{n-1} p^{n-1} q + \cdots + a_1 p q^{n-1}}_{} + a_0 q^n}^{\text{divisible by } p} = 0$$

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$$\text{where } \overbrace{a_n p^n + a_{n-1} p^{n-1} q + \cdots + a_1 p q^{n-1}}^{\text{divisible by } p} \quad \text{and} \quad \underbrace{a_{n-1} p^{n-1} q + \cdots + a_1 p q^{n-1} + a_0 q^n}_{\text{divisible by } q}$$

Dead On or Completely Off Base

Corollary: A root of monic polynomial is either an integer or an irrational.

Because $q \mid 1$ it, follows $q = \pm 1$.

Since 2 is not a square, $\sqrt{2}$ is a non-integer root of the monic polynomial $x^2 - 2$ and hence is irrational.

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