Spring 2014 Welcome

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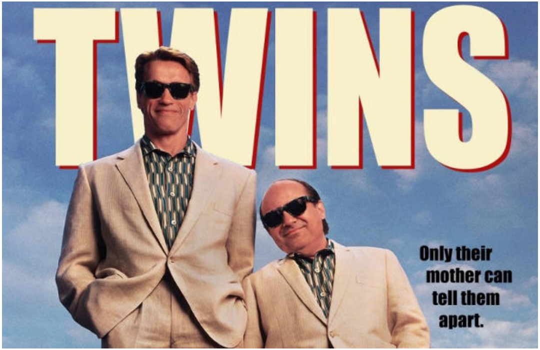
Spring 2014
Florida State University, Tallahassee, FL
Jan 3, 2014

Students: The Good and the Bad.

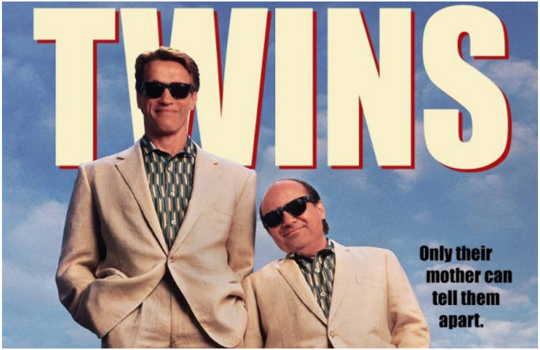


The 1988 Movie: Twins

A physically perfect but innocent man goes in search of his long-lost twin brother, who is a short small-time crook.

There is a sequel planned called Triplets, adding Eddie Murphy.

The good, the bad and the ugly is a 1966 Clint Eastwood Movie.



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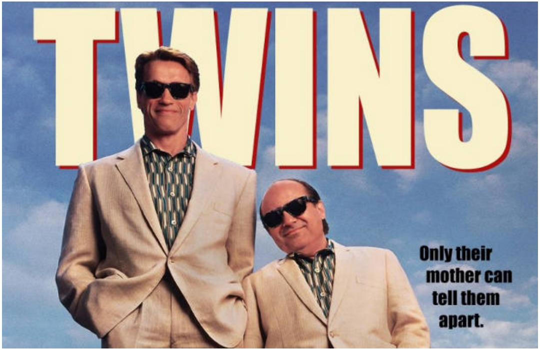

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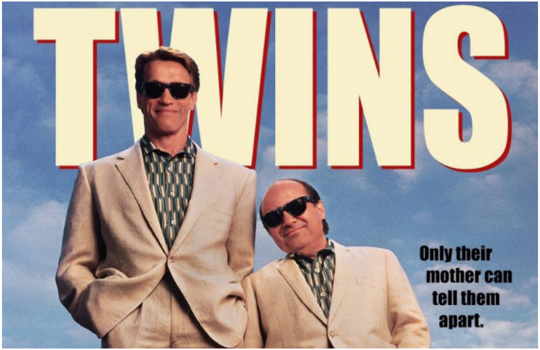

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Students: The Good

You are the math department, most students do not see a research professor. Be professional.

You have a support system, problems can be refered to course coordinators, to mentors or to me.

You serve the students best by giving reasonable and fair grades. Don't give away grades, the expectation is that somewhere between 10-25% of the students should earn A's.

Email: The Good and the Bad

The good: You don’t have to answer email. Just forward requests to add your class to Pamela. If the question is answered on the web page (or syllabus), reply “isn’t your question answered on the web page (or syllabus)”.

The bad: The students might not read email without encouragement. Even worst, they will reply to your email, asking a question which was answered in your included message. Reply: “read my email carefully.”

Disabilities: The Good and the Ugly

The good: The SDRC is the sole place to decide if a student might need an accommodation. You don't need to learn details, nor make the judgement of the maximal accommodations a particular student can ask for. The request of extra time on tests is easy to agree to, too; just make them take the test in the SDRC.

The ugly: The SDRC letter is just the starting point for discussion, it itself is not the request. Suggestion: do not accept a letter in the classroom, ask the student to bring it to your office. Discuss their requests. Unlimited excused absences?

Analysis: The Real and the Complex

In Real Analysis counter-examples rule, there are few univerisal theorems.

In Complex Analysis universal theorems rule, there are few counter-examples.

The sequel?: the Complex, the Real and the Numerical.

$g(x)$, a function considered by Euler

$$g(x) = \int_0^\infty \frac{e^{-t}}{1+xt}\,dt$$

If $x < y$, then $1/(1+xt) > 1/(1+yt)$ so $g(x) > g(y)$ and g is monotone decreasing. Eventually all derivatives of $g(x)$ are monotone. (Alternating between decreasing and increasing.)

It is a nice function, $g(x)$ is C^∞ for $x \in [0,\infty)$ and

$$g^{(n)}(0) = (-1)^n (n!)^2$$

The Taylor series for $g(x) = \sum (-1)^n n! x^n$ which diverges for all non-zero x.

Using Cauchy principle values, $g(x)$ is also defined for $x < 0$.

A Lemma

By induction, using integration by parts:

$$\int_0^\infty t^n e^{-t}\, dt = n!$$

Let $u = t^{n+1}$ and $dv = e^{-t}$

$$\int_0^\infty t^{n+1} e^{-t}\, dt = -e^{-t} t^{n+1}\Big|_0^\infty - \int_0^\infty - e^{-t}(n+1)t^n\, dt$$

$$= 0 + (n+1)n!$$

Derivatives of $g(x)$

If $h(x) = \int_a^b f(x,t)\,dt$, then

$$h'(x) = \int_a^b \frac{\partial f(x,t)}{\partial x}\,dt$$

$$g^{(n)}(x) = \int_0^\infty \frac{(-1)^n n! t^n e^{-t}}{(1+xt)^{n+1}}\,dt$$

$$g^{(n+1)}(x) = \int_0^\infty \frac{(-1)^{n+1}(n+1)! t^{n+1} e^{-t}}{(1+xt)^{n+2}}\,dt$$

$$g^{(n)}(0) = \int_0^\infty (-1)^n n! t^n e^{-t}\,dt = (-1)^n (n!)^2$$

$g(z)$, a multi-valued complex function

$$g(z) = \int_0^\infty \frac{e^{-t}}{1+zt}\,dt$$

If the closed curve $C \subset \mathbb{C} \backslash (-\infty, 0]$, then $\int_C g(z)\,dz = 0$ so $g(z)$ is analytic with a branch cut at $(-\infty, 0]$

$$\int_C g(z)\,dz = \int_C \int_0^\infty \frac{e^{-t}}{1+zt}\,dt\,dz$$

$$= \int_0^\infty \int_C \frac{e^{-t}}{1+zt}\,dz\,dt = \int_0^\infty 0\,dt = 0$$

$g(z)$ has ∞-many values like and similar to $\log(z)$.

Borel's Theorem

For any sequence of reals $\{a_n\}$ there is a C^∞ function $f(x)$ whose Taylor's series at $x = 0$ is $\sum a_n x^n$.

This is not true for functions analytic at $x = 0$, no analytic function has Taylor series $\sum (-1)^n n! x^n$.

Borel's Theorem was proved by Peano over decade before Borel proved it.

Borel and Peano