Paradox Pair

This welcome is brought to you
by Game Theory,
Braess' Paradox,
and the Chaiman's Paradox

Steven F. Bellenot

Jan 4, 2019

The talk is a slide show. The slides are framed in yellow rectangles. The quotation that follows, is what might have been said while the audience was looking at the slide. The blue comments like this one were added later and not part of the welcome. The title frame above was not the original.

Game Theory Paradoxes and Dilemmas

Self interest can be harmful

- Prisoner's Dilemma
- Adding routes can decrease throughput
- Adding powers can result in a less desired outcome

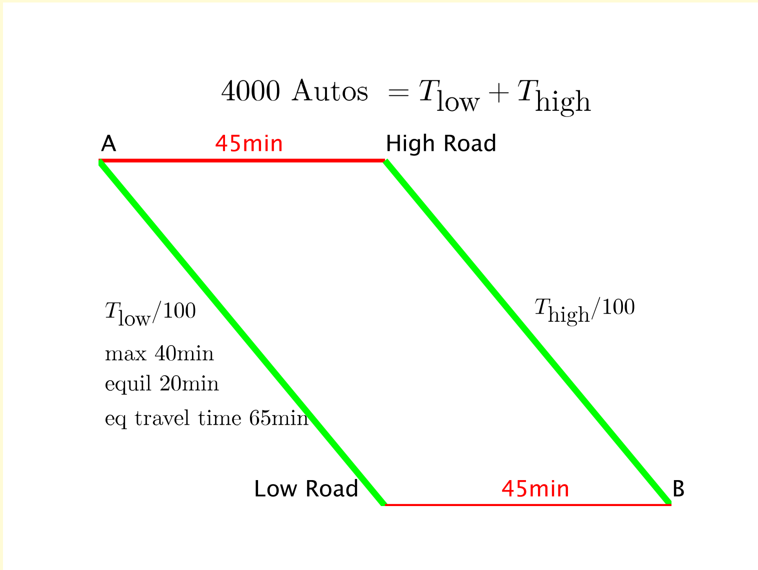
> One of the lessons of Game theory is that self interest, like in the Prisoner's Dilemma, can result in a less disirable state. I have two paradox's one that shows adding a fast shortcut can result in longer wait times. The other is giving the chairman increased powers can lead to less desirable outcomes.

The prisoner's dilemma can be represented by a matrix, the $(0, -3)$ notation, means Rose gets 0 years in prison (goes free) and Colin gets 3 years in prison. The negative number is the number of years in prison.

		Colin	
		Silent	Rat
Rose	Silent	(-1, -1)	(-3, 0)
	Rat	(0,-3)	(-2,-2)

Note no matter which row Rose picks, Colin is better off ratting on her. If Rose is silent, ratting sets Colin free, which is better than one year in prison. If Rose rats on Colin, rattin on Rose reduces Colin's prison time from three to two years. The same is true for Rose. But if they both stay silent, they both get 1 year in prison. (Apparently the police have them nailed for a lessor offense.)

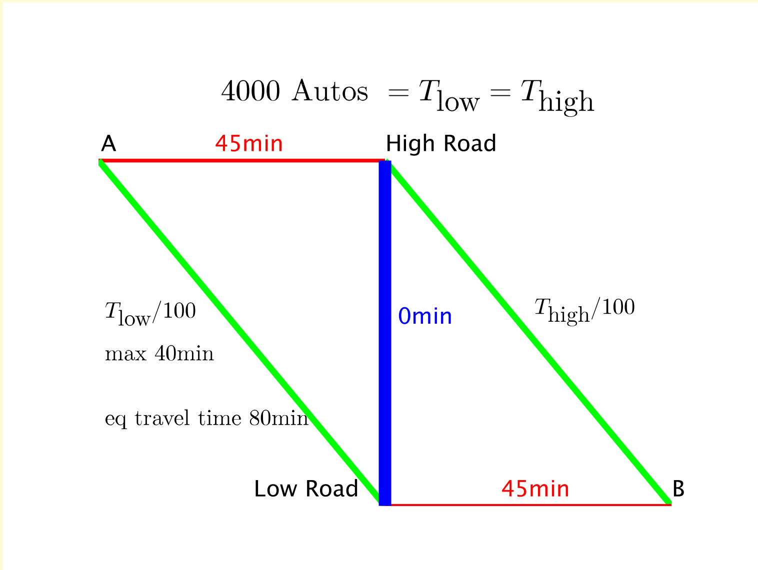
A Nash Equilibrium Traffic Flow



We are looking at two routes from A to B. The red segments take 45 minutes, not matter how much traffic. The time on the green seqments depend on the traffic. If all 4000 auto's take the low road, it will take each of them $4000/100 = 40$ minutes. But the Nash Equilibrium, divides the autos into two groups of 2000, half going the low road and half going the high road. The green segments take 20 minutes and the travel time is 65 minutes. There is no advantage for changing routes.

If one of the low roaders swicthes to the high road. His travel time goes from 65 minutes to 65.01 minutes.

The Short Cut that takes 15 minutes longer



Suppose a high speed route, shown in blue is added. Now all the autos, take the green, blue, green route, for a total of 80 minutes. No one takes a red route has it adds 5 minutes to route. This was actually noted in real life, both in Seoul, Korea (removing a motorway) and in New York City (temporary closing 42nd Stree).

This is called Braess' Paradox.

A Three-Way Tie

Members X (Chair), Y and Z make up a hiring committee. There are three candidates a, b, c:

Member	Perferencs
X	$a > b > c$
Y	$c > a > b$
Z	$b > c > a$

Give the Chair the power to break ties.
Naive: Everyone picks their favorite, then X breaks the tie selecting a, Z's least favorite.

We have X, Y, Z on the hiring committee and three candidates a, b, c and each has a different perference. If everyone votes for their favorite we have a three way tie. Suppose the Chair X is given the power to break ties, then a would win. Note a is Z's least favorite.

This has happened. The Chair doesn't have this power, but the other members might defer to the Chair.

X's strategy

If Y and Z pick the same candidate, X's vote is irrelevant.

If Y and Z split their vote, X votes for a and either a wins outright, or by producing a three way tie, which is decided by X to be a.

X can do no better than voting for a

This slide says X will vote for a, because he can do not better.

Winners assuming X votes for a

			Y	
		c	a	b
Z	b	a	a	b
	c	c	a	a
	a	a	a	a

Y votes for c and so does Z. The result is c wins. This is X's least favorite outcome.

We reduce the game assuming X votes for a. If Z picks b, Y can do no better than c. If Z picks c, Y can do no better than c and if Z picks a, Y can do no better than c. So Y picks c. Now Z is limited to the red column and also picks c. So c, X's least favorite wins.

Where did we use the Chair's extra power? When Y, Z slipt between b, c.

Chairman's Paradox

Giving more power to the Chair, results in the Chair getting his least favorite outcome.

Too sum it up, more is less.

The Take Away

Fight like hell not to be the chair.

The chair is not always a desirable position.

Being the associate chair at time, this got a good laugh.

Picture sources

Traffic pictures are by the author using Scilab