Honors Day 2010
It's Fundamental, My Dear Gauss

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Department of Mathematics
Florida State University

Honors Day
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Apr 2, 2010

Math, It's Fundamental

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- There is no royal road
- There is no math pill
- Skills are not FCAT-able
- But it is a human activity

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Fundamental Theorems

- Fundamental Theorem of Arithemetic
- Each integer $n > 1$ has a unique prime factorialization
$$n = p_1 p_2 \cdots p_k$$
with $p_1 \leq p_2 \cdots \leq p_k$
- Fundamental Theorem of Algebra
- Each non-constant polynomial has a root
- Fundamental Theorem of Calculus
- Part I: If $f(t)$ is continuous,
$$F(x) = \int_a^x f(t)\, dt$$
then $F'(x) = f(x)$

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Fundamental Theorem of Arithmetic

Euclid's Elements ($\approx$ 300 bce) contains a proof. Three of the 13 books, VII, VIII and IX are about number theory. (But Euclid had no notation for the product of more than 3 numbers.)

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Proof

Existence:
By Strong induction:
Suppose n is first integer for which existence fails. Then n cannot be prime, so $n = mk$ with $m, k < n$. But both m and k are products of primes, so n is also a product of primes.

One Cryptography Method depends on it being very hard to find m and k

Uniqueness:
Euclid's lemma: If p is prime and $p|ab$, then $p|a$ or $p|q$.

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Uniqueness Proof Cont

Suppose not:
find the smallest counterexample with

$$p_1 p_2 \cdots p_k = q_1 q_2 \cdots q_m$$

If follows that $p_i \neq q_j$,
or we could cancel from both sides yielding a smaller counterexample.
We can assume $p_1 < q_1$.
By Euclid's lemma $p_1 | q_j$ for some j.
Which implies q_j is not prime.
A contradiction, so there is no counterexample.

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- James Gregory (1638-1675) proved a restricted version
- Isaac Barrow (1630-1677) proved a general version, geometric proof without using limits.
- Isaac Newton (1643-1727) Barrow's student, developed limits and wanted to call the subject:
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Fundamental Theorem of Calculus, Part 2

- Second part Fund Thm: If F is an anti-derivative of f then,

$$\int_a^b f(t)\,dt = F(b) - F(a)$$

(Newton-Leibniz Axiom)

- the 2nd part is stronger as f does not have to be continuous. $F(x) = x^2 \sin(1/x)$ and $F'(x) = f(x) = x \sin(1/x) - cos(1/x)$ but $F'(0) = 0$.
- Current textbooks use Riemann integration Riemann (1826-1866) was a student of Gauss.

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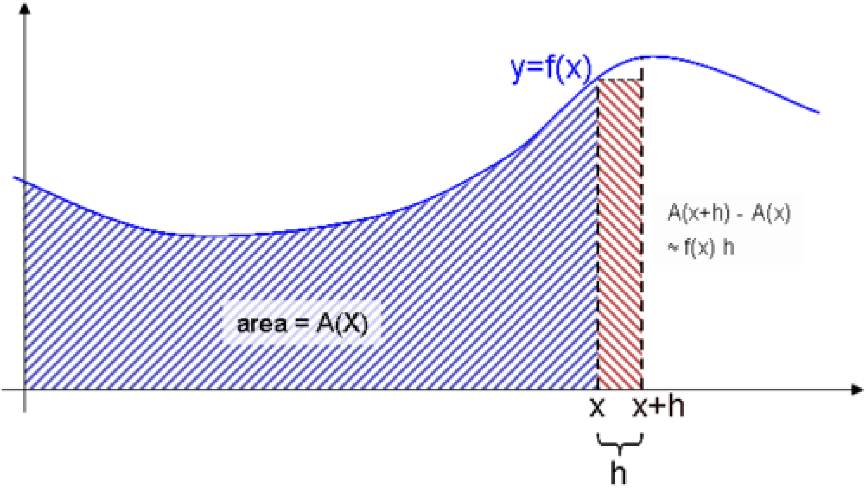
Proof of Part I



Fundamental Theorem of Algebra

Every non-constant polynomial with complex coefficients has a complex root.

Known in some form to Rother 1608 (may have n solutions) and Girard 1629 (has n solutions, but not all polynomials). Many attempted proofs: D'Alembert 1746 Euler 1749, de Foncenex 1759, Lagrange 1772, Laplace 1795 assumed the existence of a root and then showed it was a complex number. Gauss 1799 gave a geometric proof in his PhD thesis. Argand 1806 and Gauss 1816 are rigorous by today's standards. Weierstrass 1891 gave a constructive proof.

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Neither Algebra nor Fundamental?

Almost all proofs require some analysis and the fastest proofs use analytic function theory. A proof must use the completeness of the reals.

Nor is the theorem fundamental for modern algebra. There are other fundamental theorems in algebra: FT of Galois, FT on homomorphisms, FT of finitely generated abelian groups.

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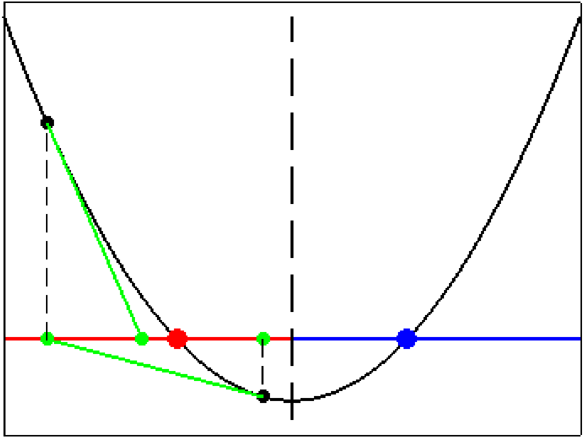
Newton's Method



Constructive Proof

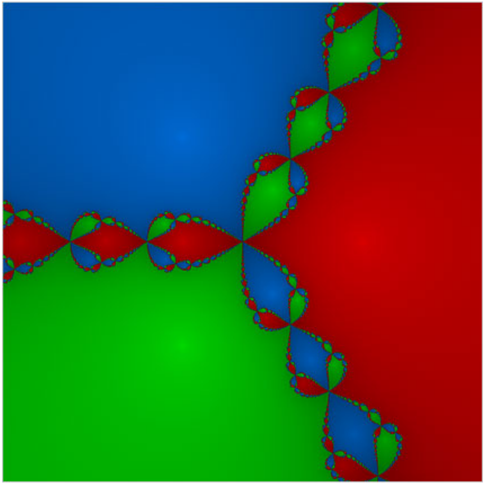
Start almost anywhere z_0 in the complex plane, repeat Newton's method

$$z_{n+1} = z_n - P(z_n)/P'(z_n)$$

and the sequence (z_n) will converge to a root of $P(z)$
If $P(z) = z^3 - 1$ with roots 1, $\exp(2\pi i/3) = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$ and $\exp(-2\pi i/3) = -\frac{1}{2} - i\frac{\sqrt{3}}{2}$, then

$$z_{n+1} = (2z_n^3 + 1)/3z_n^2$$

Degree 3 implies Chaos



Other Fundamental Theorems

Ordinary Differential Equations: one might call the existence and unqueness of solutions fundamental but we don't.

Strang (1993) declared a theorem to be the Fundamental Theorem of Linear Algebra.

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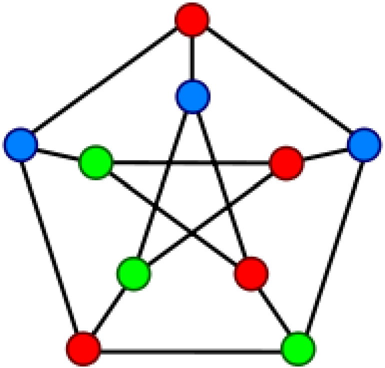
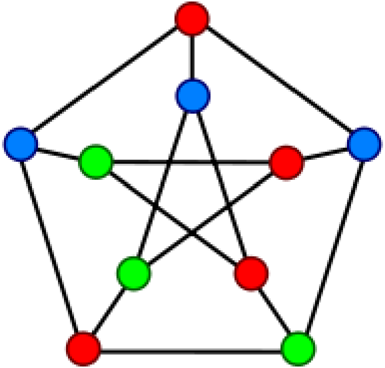
Graph Theory



The color points are **vertices**, the connecting lines are are **edges**. The degree of a vertex is the number of incident edges. Here each vertex has degree 3. This graph is the Petersen graph, the figure shows it can be 3-colored, adjacent vertices have different colors. The web is just one big graph.



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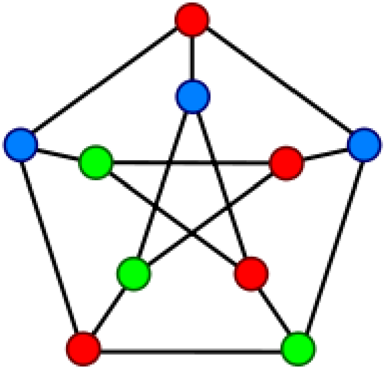

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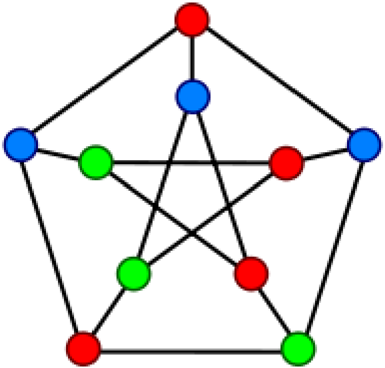

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The Handshaking Lemma

$$\sum_{v \in V} deg(v) = 2|E|$$

The sum of the degrees of the vertices is equal to twice the number of edges.

Proof: (Euler 1736) double counting. Count (v, e) where v is incident to e, two ways. Vertex v belongs to $deg(v)$ pairs while edge e belongs to 2 pairs, one for each vertex.

This is the first paper on Graph Theory. It could easily be elected to being the fundamental theory of graph theory (or topology).

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There is always an even number of odd vertices. (Proof: the RHS, $2|E|$ is even, the LHS would be odd if there were an odd number of vertices of odd degree.)

In a honor's ceremony, people shake hands, and an even number of people must have shaken an odd number of others peoples hands.

Where is Gauss?

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3. AD Michal
4. M Bocher
5. Felix Klein
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