Honors Day 2011
When Did *i* Become Respectable?

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Honors Day
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A Historical View

Math courses often diverge wildly from the Historical Development of Mathematics.

A classic example is the concept of zero 0 as a symbol which allows the current decimal notation of numbers and our current system of arithmetic. The Romans used Roman numerials and tools like an abacus. The 6th century zero from India arrived in Europe in the 11th century.

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A classic example is Leibnitz notion of infintesimals (1684,1686). Infintesimals made Calculus accessible. Principia (1687) Newtons methods fluxions (written in 1671 but published 1736) basically kept Britian a half a century behind the continent.

But infintesimals were replaced by limits that were defined via the ϵ and δ method of Cauchy (1821) and Weierstrass (1863)

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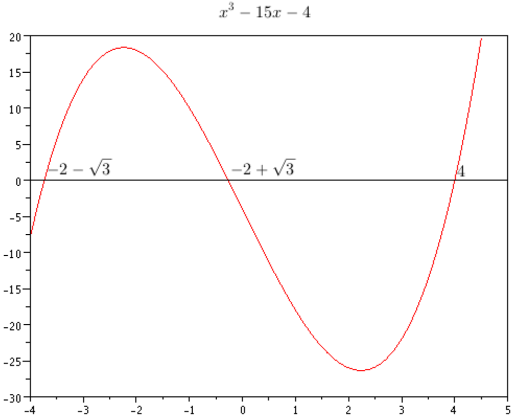
A cubic with 3 real roots



Let The Use Begin

Girolamo Cardano (1501-1576) published The Great Art in 1545

- FOUND REAL ROOTS roots of the cubic polynomial using square roots of negative numbers.
- Solved cubic and quartic polynomials
- $x + y = 10, xy = 40$ and obtains $x = 5 + \sqrt{-15}, y = 5 - \sqrt{-15}$
  "This subtility results from arithmetic of which this final point is as I have said as subtile as it is useless." Cardano perplexed.
- Bombelli 1572 has all the algebra of complex numbers.

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Solving $x^3 + mx = n$

$$(a-b)^3 + 3ab(a-b) = a^3 - b^3$$

$$3ab = m \qquad a^3 - b^3 = n \implies (a-b) \text{ solves } x^3 + mx = n$$

$$b = m/3a \implies a^3 - m^3/27a^3 = n$$

$$a^6 - na^3 - m^3/27 = 0 \quad \text{quadratic in } a^3$$

solve for a^3 take cube root

Solve for b using $b = m/3a$, and obtain a root $a - b$

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$$b = -15/3a = -5/a, \quad a^3 + 125/a^3 = 4$$

$$a^6 - 4a^3 + 125 = (a^3 - 2)^2 + 121, \quad a^3 = 2 + 11i$$

a	b	$a - b$
$2 - i$	$-2 - i$	4
$-1 + \frac{\sqrt{3}}{2} + i(\frac{1}{2} + \sqrt{3})$	$1 - \frac{\sqrt{3}}{2} + i(\frac{1}{2} + \sqrt{3})$	$-2 + \sqrt{3}$
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René Descartes (1596-1650) published The Geometry in 1637

- Gave $\sqrt{-1}$ the derogatory name imaginary.
- Invented Analytic Geometry
- Invented Cartesian Coordinates
- A true synthesis of algebra and geometry
- Constructed lines (instead of rectangles) of length ab
- Solved quadradic equations by construction.

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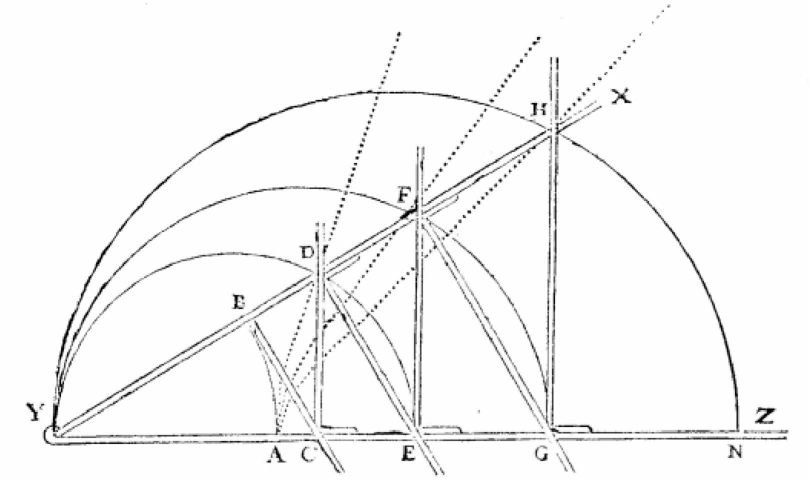
Cardano and Negatives

None of the coefficients in Cardano's polynomials were negative, he consider the polynomials below as different "cases"

$$x^3 + 125x = 4$$
$$x^3 = 125x + 4$$

Descartes and Negatives



Trigonometry and Complex

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Euler, Lagrange and D'Alembert were adept at complicated manipulations of algebraic equations that included the square roots of negative numbers, while insisting at all times such magnitudes were "impossible"

All such expressions as $\sqrt{-1}, \sqrt{-2} \ldots$ are consequently impossible or imaginary numbers, since they represent roots of negative quantities; and of such numbers we may truly assert that they are neither nothing, nor greater than nothing, nor less than nothing, which necessarily constitutes them imaginary or impossible. (1770)

$1 = \sqrt{1} = \sqrt{(-1)(-1)} = \sqrt{-1}\sqrt{-1} = i^2 = -1$

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$\sqrt{ab} = \sqrt{a}\sqrt{b}$ requires non-negative a and b.

Gauss (1777-1855)

These are not “impossible” magnitudes, but “shadows of shadows”.
(Shadow is the projection onto the real line, in particular of an object rotating on the unit circle.)
The true sense of the square root of -1 stands before my mind fully alive, but it becomes very difficult to put it in words. (1811)

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