A Simple Ecological Model

Steven F. Bellenot

Department of Mathematics
Florida State University

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Ecology

1. The branch of biology that deals with the relations of organisms to one another and to their physical surroundings.
2. the study ot the interaction of people with their environment.
3. the political movement that seeks to protects the environment, especially from pullution.

The FSU Mathematical Environment

The Undergraduate Math Majors:

- Pure Math
- Applied Math
- Actuarial Science
- BioMathematics
- FSU-Teach/Mathematics

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Why come to FSU for Mathematics?

- Access to Research as an Undergraduate
- Access to Graduate Classes
- Depth of the faculty and computer resources
- Upper level classes are small

- Created with a Hughes Grant with the goal of teaching more math to biologists and perhaps more biology to mathematics students.
- Uses the power of mathematical software (matlab, maple) to reduce the mathematical pre-requisite to only one semester of Calculus.
- Why Mathematics is Biology's next microscope.
- Why Biology is Mathematic's next Physics.

MAP 2480 Students

- Course is a 1-hour MAP 2480 Biocalculs Lab
- Biology majors, many Pre-Med, with a few biomath majors.
- Calculus I: MAC 2311 pre-requisite
- Some students are 3-4 years from Calculus
- Pre-meds work for grades, cram instead of learning
- No programming experience

Mathematical Modeling

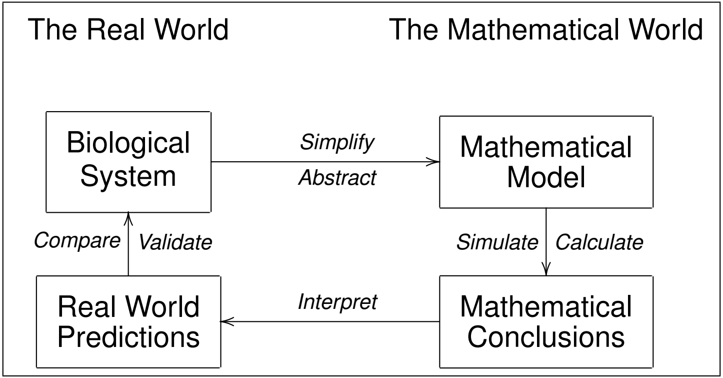

Figure: The Modeling Process Cycle

Teaser Question for this Ecology Lab

- Why are there so many different species?
- We give evidence as to why this is a hard question, we don't answer it.
- In fact, for each kind of resource, there is only one species using that resource.
- Diversity of species implies diversity of resources.

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Two more terms

- *Spacial:* of or relating to space.
- *Stochcastic:* randomly determined; having a random probability distribution or pattern that may be analyzed statistically but may not be predicted precisely.
- *Curve Fitting:* the process of constructiong a curve or mathematical function that has the best fit to a series of data points.

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Life on a checkerboard

- Plants grow on a checkerboard
- Each square has one plant
- Reproduction via nearest neighbors
- Repeat:
  - select plant
  - Voter Model, plant is replaced by a neighbor's offspring
  - Invasion Process, plant's offspring replaces a neighbor
- generation is nm births for $n \times m$ checkerboard.

The Colormap

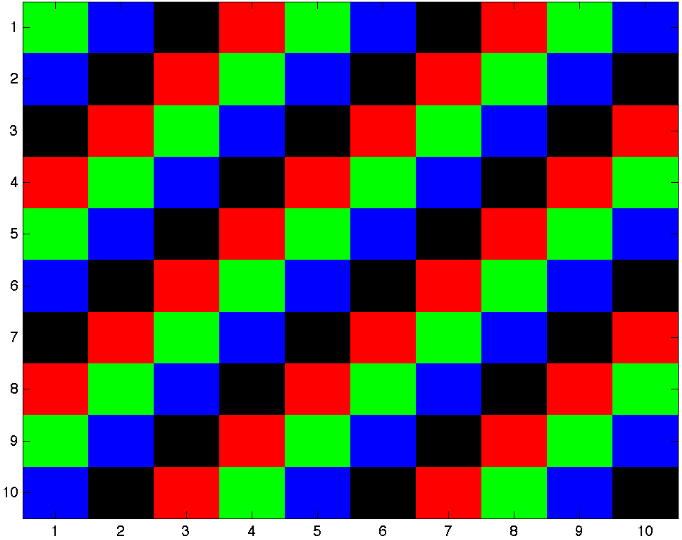
Species	Image Color
1	black
2	red
3	green
4	blue
5	yellow
6	magenta
7	cyan

The colormap installed by init.m; (careful 0 is also black).

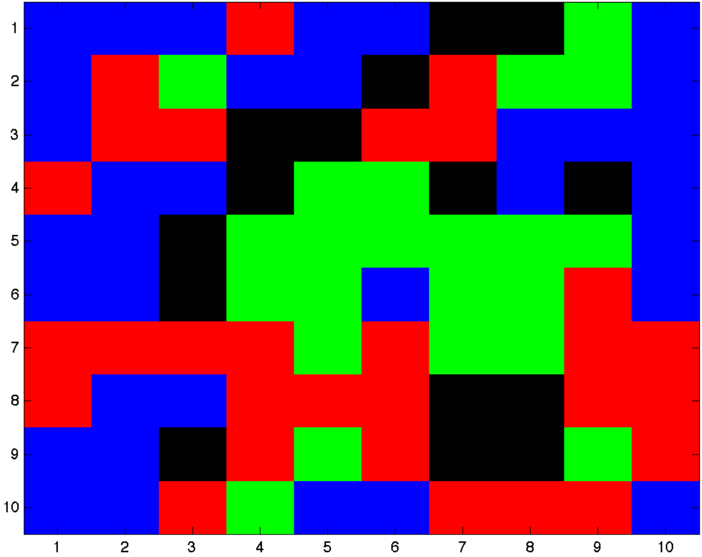
Initialization Code

```
n=10; A = zeros(n,n);
for i=1:n, for j=1:n,
A(i,j) = mod(i+j,4)+1;
end; end;
cm = [ 0 0 0; % 1 black
1 0 0; % 2 red
0 1 0; % 3 green
0 0 1; % 4 blue
1 1 0; % 5 yellow
1 0 1; % 6 magenta
0 1 1 ]; % 7 cyan
colormap(cm);
% 0 also gets black (why the plus one)
```

Initial Position



One generation later



Sample Code

```
init;
image(A); % Scilab uses Matplot
rand('seed', 1234);
A = generationip(A);
image(A);
```

To get a movie:

```
init; for i=1:100;
A=generationip(A); image(A); pause(0.01);
end;
```

Observations

Experience with this model shows two behaviors:

1. The colors blotch together; plants of the same color tend to clump or cluster together.
2. Eventually all rectangles have the same color, one species wins and the others die out.

The second model

Suppose there were at most 2 species and we consider only the population of species 1. Each event in our old model did one of there things:

1. A plant was replaced by the same species. Popuation change: 0
2. A plant of species 1 was replaced by species 2. Population change: -1
3. A plant of species 2 was replaced by species 1. Population change: +1

And the last two events are equally likely. (Chose the edge and the then the direction.)

Dunkard's walk on 2×2

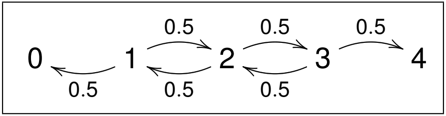


Figure: Diagram showing the transitions with probabilities

$$T = \begin{bmatrix} 1 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0 & 0 \\ 0 & 0.5 & 0 & 0.5 & 0 \\ 0 & 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 1 \end{bmatrix}$$

If $X = \begin{bmatrix} x_1 & x_2 & x_3 & \dots & x_n \end{bmatrix}$ with $x_i \geq 0$ and $\sum x_i = 1$, then as $n \to \infty$, $T^n X \to \begin{bmatrix} s & 0 & 0 & \dots & 1-s \end{bmatrix}$ some s, $0 \leq s \leq 1$.
Fixed points of T, eigenvectors for the eigenvalue 1.

Lab Questions

1. For loop output (for $i = a : b$, i^n, end;)
2. Next random number after seeding
3. Who is the winner after seeding
4. The movie – oracle question
5. Two steps for the dunk
6. 3 or 4 for the dunk
7. limit $T^n X$
8. Movie 2: 25x25 and red vs green – oracle question

Oracle Questions

Often we will have complex commands for you to do which are not easily graded on the computer. After you have done the task, as one of the instructors to check your work. If it is correct he will give you the answer to question "What is the answer to question number a?"

Answer: There is a sheet with the answers. The answer is the next random number for a simple-to-compute random number generator.

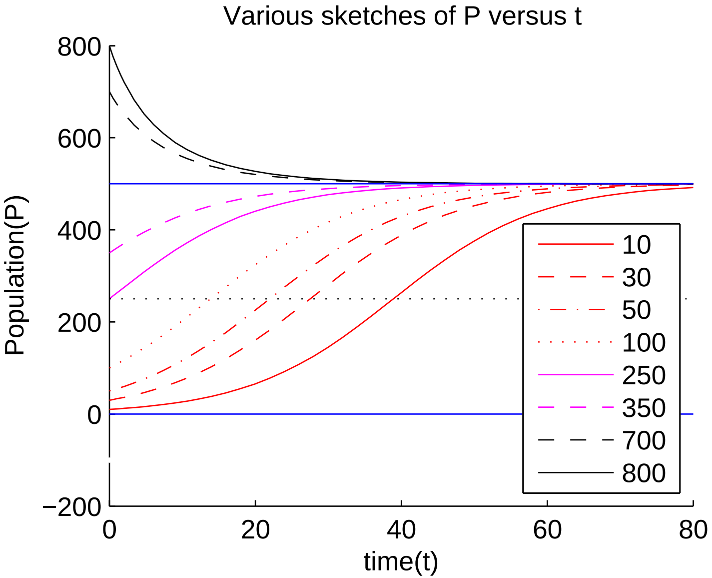
Additional Directions

- Monte Carlo simulations to estimate time to domination.
- Geographic shapes (islands and land bridges) instead of checkerboards.

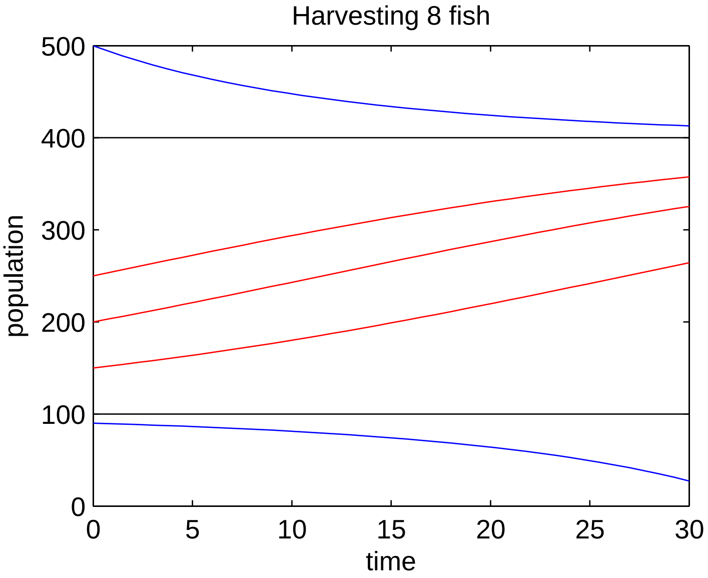
Other Ecological Labs

- Logistic Model of Population Growth
- Logistic Model of Population Growth plus Harvesting
- SIR epidemic model
- Predator Prey Model

Logistic Growth



Logistic Growth with Harvesting



SIR Epidemic Model

- In a total population of n individuals there are, at any time t,
- $x(t)$ members of the population who are susceptible to a contagious disea se,
- $y(t)$ infectious carriers of this contagious disease,
- $z(t)$ individuals who are recovered and immune.
- It is clear $x(t) + y(t) + z(t) = n$ for all $t \geq 0$.

$$\frac{dx}{dt} = -\lambda x(t)y(t)$$
$$\frac{dy}{dt} = \lambda x(t)y(t) - \mu y(t)$$
$$\frac{dz}{dt} = \mu y(t)$$

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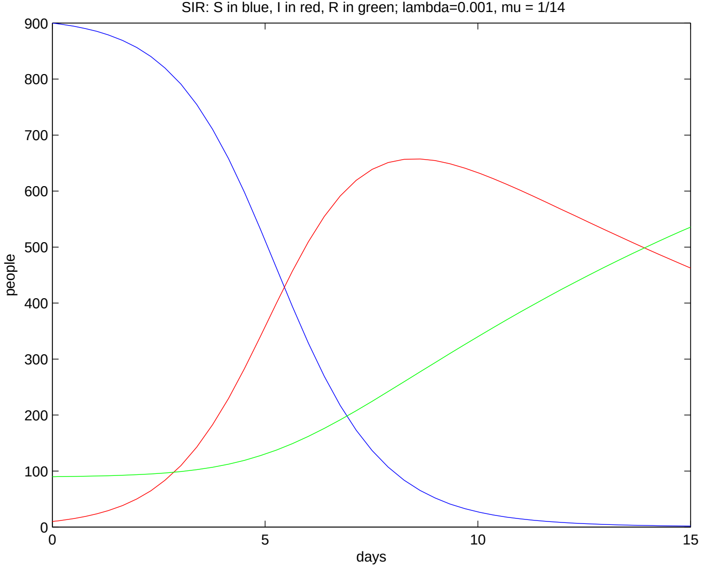
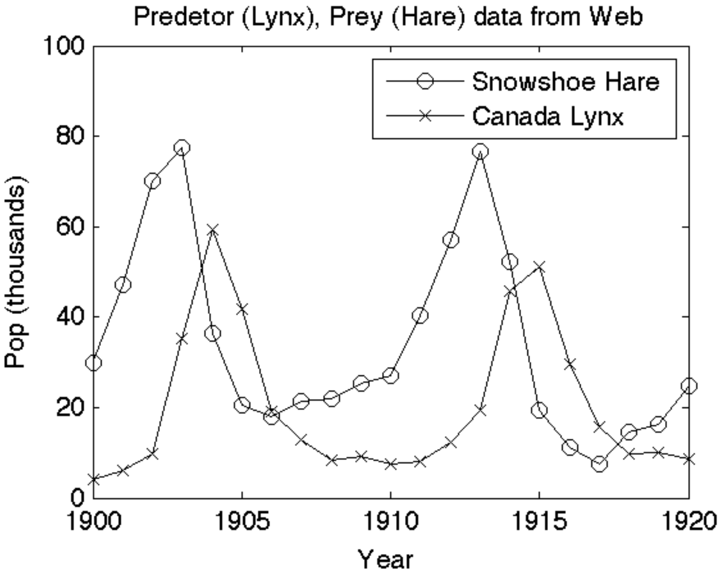


Predator Prey Model



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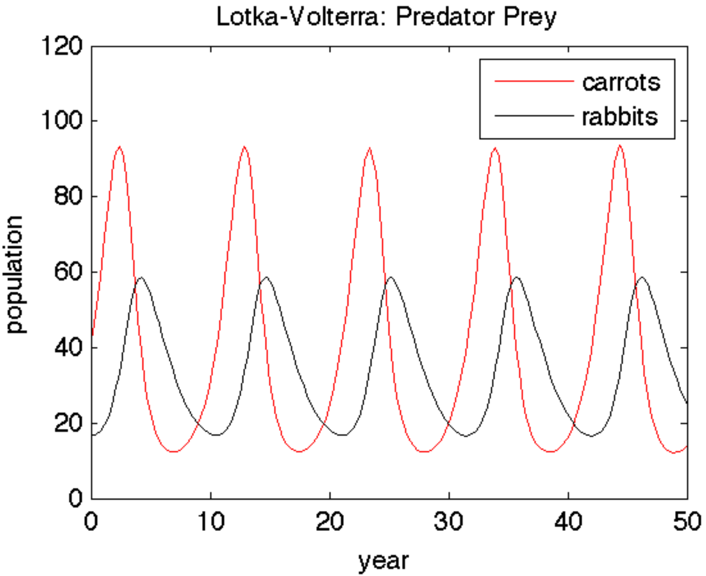

Summary

- Stochastic and spacial models provides a more general first view of modeling as compared to curve fiting.
- A nice animation to amuse the students.
- A dunkards walk, monte carlo, islands, and more.

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