Friend Networks on Convex Polynomials or 1, 2, 3, 4, ?

This welcome is brought to you
by Social Networks,
Food Web Networks,
and Convex Polyhedra

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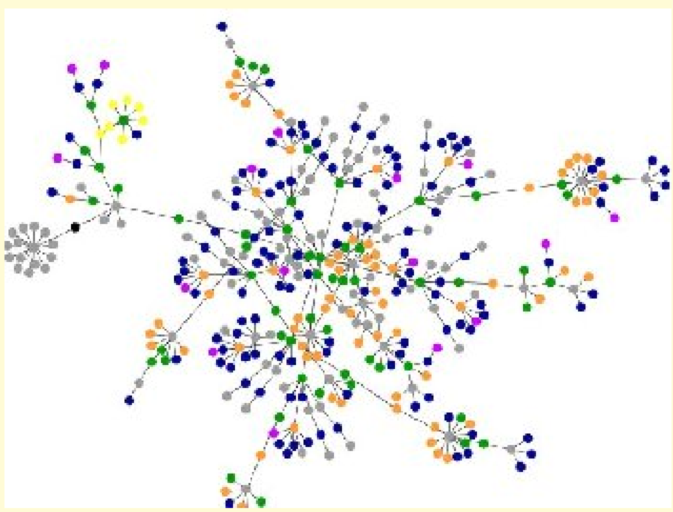
The talk is a slide show. The slides are framed in yellow rectangles. The quotation that follows, is what might have been said while the audience was looking at the slide. The blue comments like this one were added later and not part of the welcome. The title frame above was not the original.

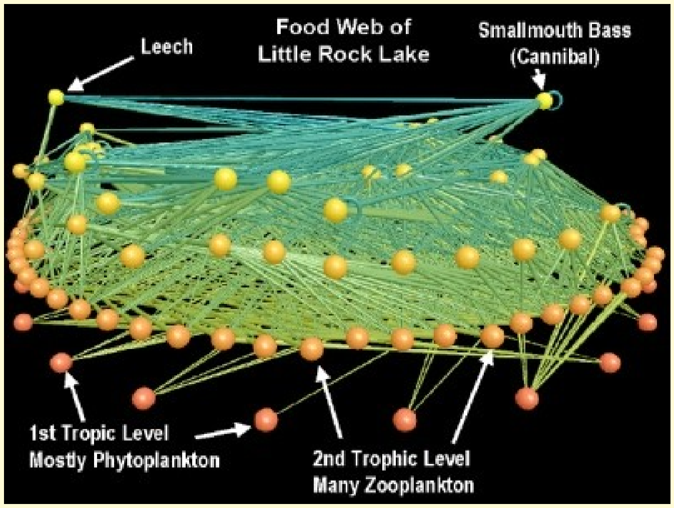
Social Networks



Why did I do social networks and food webs?

There was a talk on big networks the semester before.

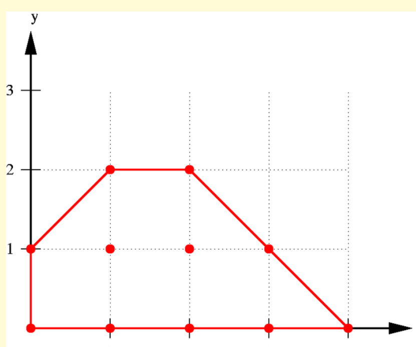
Food Web Networks



Little Rock Lake is in Wisconsin. Note the loop, making the smallmouth bass a cannibal. My food web section in biocalculus, had no cannibals. An example where not all edges are friendly.

new stuff

Convex Polyhedra



A *convex polyhedra* is the convex hull of a finite set of points in $\mathbb{R}^n$. Network *vertices* are the extreme points (i.e. $(2,2)$ and $(0,4)$ but not $(1,1)$ nor $(3,1)$) and Network *edges* are extreme line segments between vertices (i.e $[(2,2),(0,4)]$ but not $[(0,0),(2,2)]$). Two vertices are *friends* if they are joined by an edge.

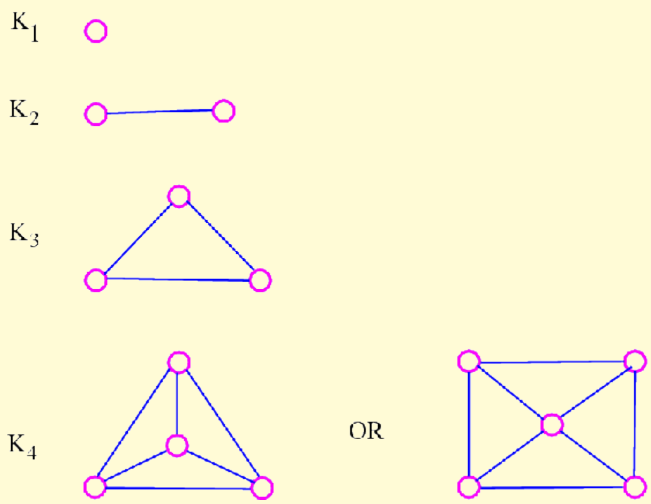
A convex polyhedra is the convex hull of its extreme points. We make the extreme points vertices, and connect vertices that share an extrama edge. In two dimensions, this will be a convex polygon. In three dimensions a polyhedra. This picture tries to clarify.

Friends mean adjacency.

Maybe I should delete the first two examples an start with a docedahedran? A cube?

Clicks

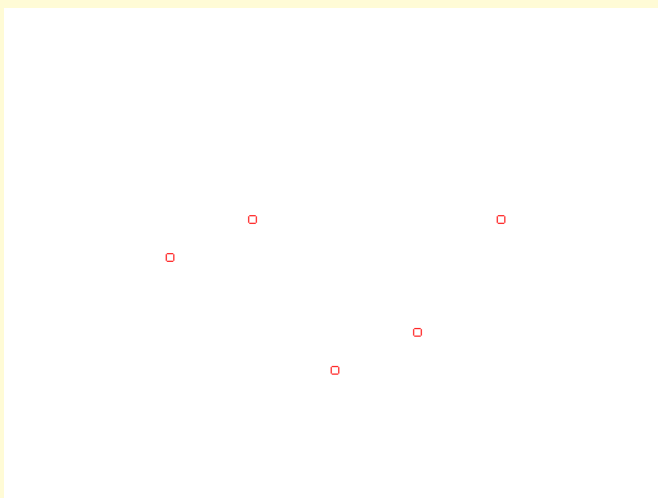
A *click* is a set of vertices, every pair of which are friends.



How are the two graphs for K_4 the same? I think they are two attempts at getting two dimensional graphs with a click of four. Both of course fail. The first failure works as a click in three dimensions, the second failure doesn't work in three dimensions.

new stuff

Max Convex Click Size in 2-Dimenisions is 3

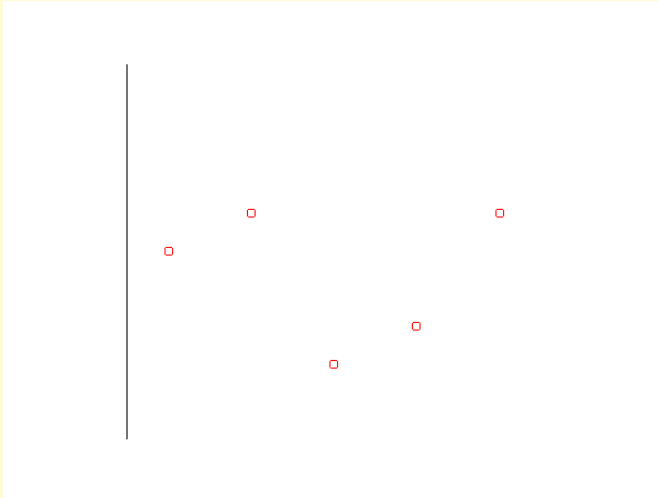


Suppose this collection of points is a max convex click

Suppose the convex hull of these points in the plane form a click.

Change the plot to larger solid dots. We want to show that the click has at most three vertices.

Max Convex Click Size in 2-Dimenisions is 3

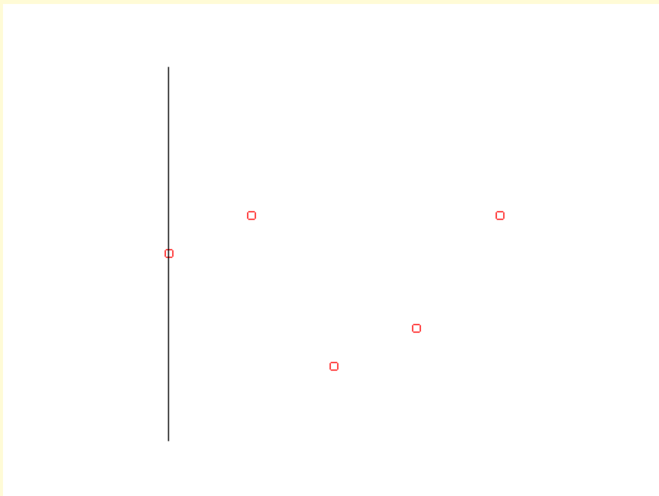


Find the leftmost point

We start by moving a line from the far left (west) towards the east. We stop when we make contact with a vertex.

I could touch more than one vertex, if so, pick the topmost.

Max Convex Click Size in 2-Dimenisions is 3

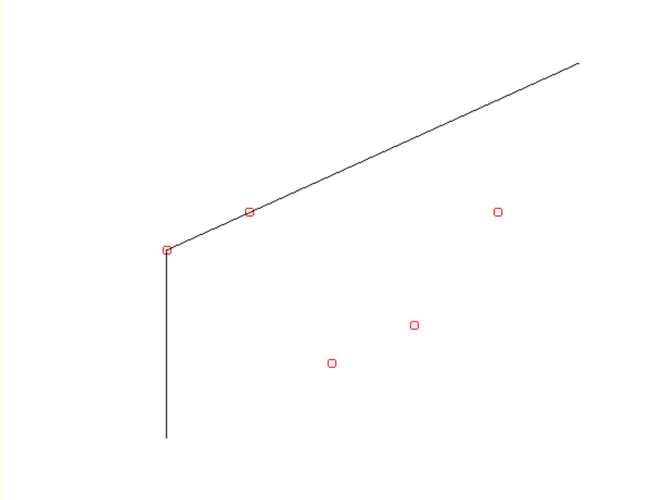


Find the next point clockwise

We are rotating the top ray and stop when we come to be in contact with a second point.

There cannot be two new points on the line. The middle one would not be an extreme point.

Max Convex Click Size in 2-Dimenisions is 3

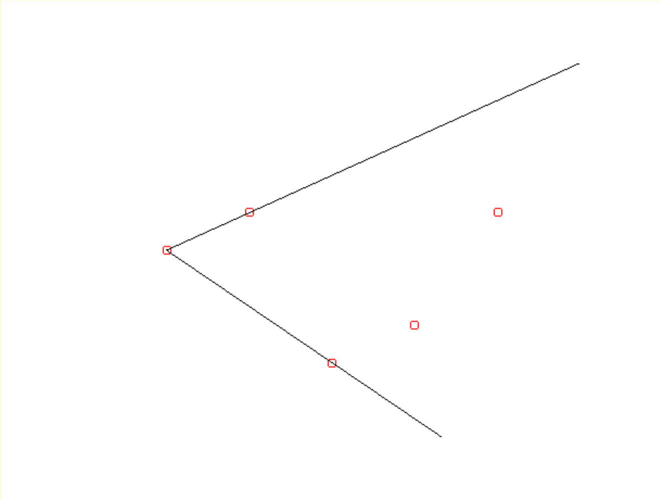


Find the next point counterclockwise

Repeat with the lower ray. We now have a angle of extreme points. All other extreme points most be inside the angle. So edge from our leftmost point, to any of the other points can be extremal. The click is at most three.

new stuff

Max Convex Click Size in 2-Dimenisions is 3

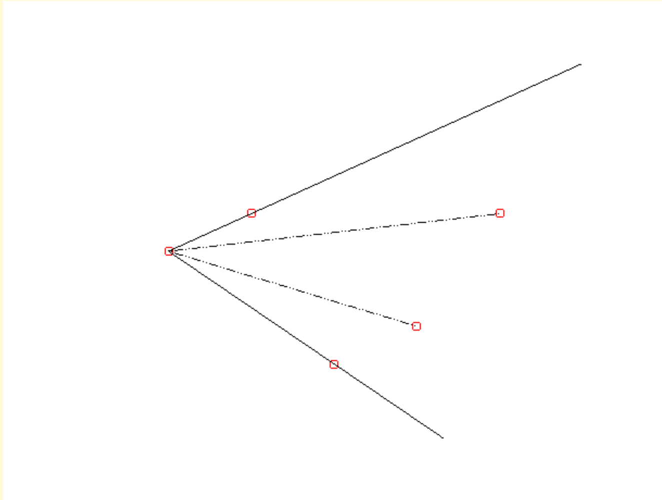


Now look at the remaining points

That is what I said.

new stuff

Max Convex Click Size in 2-Dimenisions is 3



They are not extremal, so max is 3.

That is what I said.

new stuff

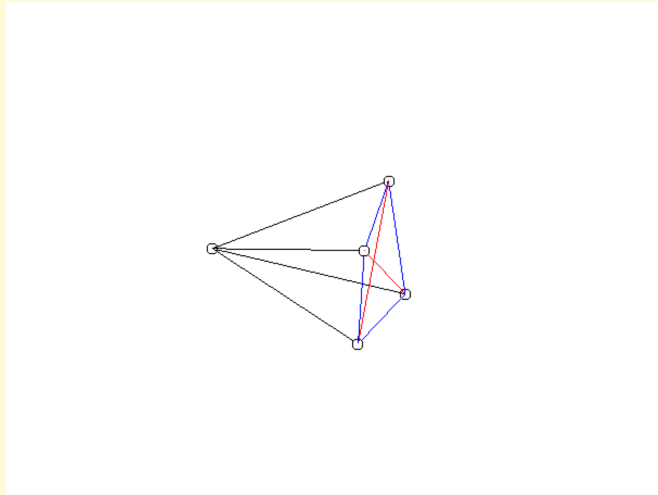
Max Convex Click Size in 1-Dimenision is 2

Only two vertices, leftmost and rightmost.

The points between the leftmost and the rightmost are not extreme points. The max click size in one dimension is 2.

new stuff

Max Convex Click Size in 3-Dimenision is 4



The tetrahedron the best you can do. This picture follows our two ideas. Move a plane from the far left. rotate it til it touchs a second point. Use the edge between them as a hinge to find the thrid point.

We can continue doing planes until we have a solid angle. This solid angle cannot have four sides because both of the red edges cannot be extrema.

new stuff

Maximal Convex Clicks Size in Dimension n

Dimension	Max Convex Click
0	1
1	2
2	3
3	4
4	?

Here is a table summarizing the results so far. We can move to four dimensions, what do we expect the answer to be?

new stuff

Maximal Convex Clicks Size in Dimension n

Dimension	Max Convex Click
0	1
1	2
2	3
3	4
4	∞

The answer is infinity!

new stuff

In 4-dimensions there is no max click size

Pick distinct t_i, $1 \le i \le n$.
The points $P_i = (t_i, t_i^2, t_i^3, t_i^4)$, $1 \le i \le n$ form a convex click.

$$f(t) = (t - t_3)^2 (t - t_5)^2 = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4$$

Note $f(t_3) = f(t_5) = 0 < f(t_i)$ for $3 \ne i \ne 5$.
Note P_i dot product $[a_1, a_2, a_3, a_4] = f(t_i) - a_0$
We have found a linear functional that exposes the edge from P_3 to P_5.

The points are on a curve called the moment curve. Any two on this curve are extreme points with an extrema edge connecting them. We illustrate this with the t_3 and t_5, using the function $f(t)$. The polynomial $f(f)$ is always non-negative and strictly positive except for the zero's at t_3 and t_5.

Looking at the coefficients of $f(t)$ we can see that f is a linear functional, a dot product. The half space that f determines contains just the two points P_3 and P_5 (and the edge between them).

I learned this from a Claremont colleges colloquium. The term the speaker used was neighborly, in this case it was 2-neighborly. The result generalizes, in d-dimensions, this construction yields a $k =$ '$\lfloor d/2 \rfloor$ neighborly set. Each k vertices are on an extrema half space.

Picture sources

social I can't find again. Perhaps replace it with `http://authenticorganizations.com/harquail/2011/01/24/your-authentic-social-network-the-identity-graph/#sthash.aALsBCch.dpbs`

foodweb picture is from `https://www.researchgate.net/figure/3D-Visualization-of-the-food-web-of-Little-Rock-Lake-Wisconsin` `fig1_221000766` N D Martinez, 1991 Artifacts of Attributes? Effects of Resoluition on the Little Rock Lake Food Web, Ecological Monographs 61, 367-392.